MATH 413 HW 13

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1. Show that $\operatorname{Inn}(S_3) = \operatorname{Aut}(S_3)$ and that $|\operatorname{Inn}(S_3)| = 6$.

Solution:

2. A subgroup K of a group G is a *characteristic* if $\sigma(K) = K$ for all $\sigma \in \operatorname{Aut}(G)$. We write K char G for this. Show that if K is a characteristic subgroup, it is normal. Give an example to show the converse is false. Show that

$$A \text{ char } B \text{ char } C \implies A \text{ char } C,$$

and

$$A \text{ char } B \trianglelefteq C \implies A \trianglelefteq C.$$

Remember (but you don't have to prove this) that $A \trianglelefteq B \trianglelefteq C$ does not imply $A \trianglelefteq C$.

Solution:

3. Suppose σ is an automorphism of G such that $\sigma(x) = x^{-1}$ for all $x \in G$. Show that G is abelian.

Solution:

4. Let G be a finite group and let σ be an automorphism such that $\sigma(x) = x$ if and only if $x = e$. Show that every element of G can be written in the form $x^{-1}\sigma(x)$ for some $x \in G$.

Solution:

5. Let G be a finite group and let σ be an automorphism such that $\sigma(x) = x$ if and only if $x = e$ and such that $\sigma^2(x) = x$ for all $x \in G$. Show that $\sigma(x) = x^{-1}$ and that G is abelian.

Solution:

6. Let G be a finite group and let σ be an automorphism such that $\sigma(x) = x^{-1}$ for more than $3/4$ of the elements. Show that $\sigma(x) = x^{-1}$ for all $x \in G$ and that G is abelian. [Hint: this problem is quite hard.]

Solution: