MATH 413 HW 15

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1. Show that, if K is a field satisfying $\mathbb{Q} \subseteq K \subseteq \mathbb{Q}(\sqrt[5]{12})$, then $K = \mathbb{Q}$ or $K = \mathbb{Q}(\sqrt[5]{12})$.

Solution:

2. Show that $\sqrt{2}$ and $\sqrt{3} \in \mathbb{Q}(\sqrt{2}+\sqrt{3})$. Find a basis for $\mathbb{Q}(\sqrt{2}+\sqrt{3})$ as a vector space over $\mathbb{Q}$. Find the minimum polynomial of $\sqrt{2}+\sqrt{3}$ over $\mathbb{Q}$.

Solution:

3. Find the minimum polynomial of $\sqrt{1+\sqrt{5}}$ over $\mathbb{Q}$.

Solution:

4. Find the minimum polynomial of $i\sqrt{3}+\sqrt{2}$ over $\mathbb{Q}$.

Solution:

5. Let $F \subseteq K$ be fields and let $u \in K$ be an element whose minimum polynomial has odd degree. Show that $F(u) = F(u^2)$.

Solution: