*1. Let $a$ and $b \in \mathbb{Z}$ with $b > 0$. If 
\[ a = bq + r, \quad 0 \leq r < b, \]
then $\gcd(a, b) = \gcd(b, r)$.

Solution:

2. a. Show that if $a$ and $b$ are integers satisfying $a^2 = 2b^2$ then $a = b = 0$. Hint: Use the Fundamental Theorem of Arithmetic.

Solution:

b. Show that $\sqrt{2}$ is irrational.

Solution:

3. Let $p$ and $q$ be primes such that $p \geq 5$ and $q \geq 5$. Show that
\[ 24 \mid (p^2 - q^2). \]

This problem is a bit hard. If you can’t do it, try to show that $12 \mid (p^2 - q^2)$ and I’ll give you partial credit.

Solution: