MATH 412 HW 2: August 29, 2015

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***1.** Let a and $b \in \mathbb{Z}$ with $b > 0$. If

$$a = bq + r, \quad 0 \leq r < b,$$

then $\gcd(a, b) = \gcd(b, r)$.

Solution:

2. **a.** Show that if a and b are integers satisfying $a^2 = 2b^2$ then $a = b = 0$. **Hint:** Use the Fundamental Theorem of Arithmetic.

Solution:

b. Show that $\sqrt{2}$ is irrational.

Solution:

3. Let p and q be primes such that $p \geq 5$ and $q \geq 5$. Show that

$$24 \mid (p^2 - q^2).$$

This problem is a bit hard. If you can't do it, try to show that $12 \mid (p^2 - q^2)$ and I'll give you partial credit.

Solution: