MATH 412 HW 4: September 15, 2015

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1. Show that every element of $\mathbb{Z}_n$ is either a unit or a zero divisor but not both.

Solution:

2. Show that $1 + 3x$ is a unit in $\mathbb{Z}_9[x]$.

Solution:

3. Let $d \in \mathbb{Z}$ be a squarefree integer which is not 0 or 1. Show that

$$R = \left\{ \begin{pmatrix} a & bd \\ b & a \end{pmatrix} : a, b \in \mathbb{Z} \right\}$$

forms a ring using matrix addition and multiplication. Show that R is isomorphic to the ring $\mathbb{Z}[\sqrt{d}]$ that was given in the previous exercice set. Show that if d is not squarefree, say $d = 4$, then R is still a ring but it is not isomorphic to $\mathbb{Z}[\sqrt{4}]$ $(= \mathbb{Z}[2] = \mathbb{Z})$.

Solution: