1. Factor each of the following polynomials in \( \mathbb{Q}[x] \) into irreducibles.
   a. \( x^5 + 4x^4 + x^3 - x^2 \)
   Solution:
   b. \( 2x^4 - 5x^3 + 3x^2 + 4x - 6 \)
   Solution:
   c. \( x^5 - 4x + 22 \)
   Solution:

2. Show that \( 30x^n - 91 \), where \( n > 1 \), has no roots in \( \mathbb{Q} \).
   Solution:

3. a. Let \( F \) be a field, \( f(x) \in F[x] \) and \( c \in F \). Show that if \( f(x + c) \) is irreducible in \( F[x] \) then \( f(x) \) is irreducible in \( F[x] \). Hint: Prove the contrapositive.
   Solution:
   b. Show \( f(x) = x^4 + 4x + 1 \) is irreducible in \( \mathbb{Q}[x] \) by showing \( f(x + 1) \) is irreducible.
   Solution: