MATH 412 HW 5

BILLY BOB

1. Factor each of the following polynomials in $\mathbb{Q}[x]$ into irreducibles.

a. $x^5 + 4x^4 + x^3 - x^2$

Solution:

b. $2x^4 - 5x^3 + 3x^2 + 4x - 6$

Solution:

c. $x^5 - 4x + 22$

Solution:

2. Show that $30x^n - 91$, where $n > 1$, has no roots in $\mathbb{Q}$.

Solution:

3. **a.** Let F be a field, $f(x) \in F[x]$ and $c \in F$. Show that if $f(x + c)$ is irreducible in $F[x]$ then $f(x)$ is irreducible in $F[x]$. Hint: Prove the contrapositive.

Solution:

b. Show $f(x) = x^4 + 4x + 1$ is irreducible in $\mathbb{Q}[x]$ by showing $f(x + 1)$ is irreducible.

Solution: