# MATH 412 HW 6 

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1. Let $S$ be a ring containing $\mathbb{Z}_{6}$ as a subring. Show that the polynomial $3 x^{2}+1 \in \mathbb{Z}_{6}[x]$ has no roots in $S$.

## Solution:

2. Show that $9 x^{4}+4 x^{3}-3 x+7$ is irreducible in $\mathbb{Q}[x]$ by finding a prime $p$ such that it is irreducible in $\mathbb{Z}_{p}[x]$.

## Solution:

3. Show that the set of nonunits in $\mathbb{Z}_{8}$ is an ideal of $\mathbb{Z}_{8}$ but the set of nonunits in $\mathbb{Z}_{6}$ is not an ideal of $\mathbb{Z}_{6}$. For which $n$ do you think the nonunits of $\mathbb{Z}_{n}$ are an ideal? Just make a guess-you do not need to prove it.

## Solution:

4. Let $I$ and $J$ be ideals of a ring $R$.
a. Show that $I \cap J$ is an ideal of $R$.

## Solution:

b. Let $I+J=\{a+b: a \in I, b \in J\}$. Show that $I+J$ is an ideal of $R$.

Solution:
c. Let $d=\operatorname{gcd}(a, b)$ in $\mathbb{Z}$. Show that $(a)+(b)=(d)$.

Solution:

