1. Let $I$ be an ideal of a ring $R$ and let $S$ be a subring of $R$. Show that $I \cap S$ is an ideal of $S$.

**Solution:**

2. Let $S$ be the set of matrices of the form \[
\begin{pmatrix}
 a & b \\
 0 & c
\end{pmatrix}
\] and let $I$ be the set of matrices of the form \[
\begin{pmatrix}
 0 & b \\
 0 & 0
\end{pmatrix},
\] where $a$, $b$ and $c$ are in $\mathbb{R}$.

   a. Show that $S$ is a ring and that $I$ is an ideal of $S$.

**Solution:**

   b. Show that $S/I \cong \mathbb{R} \times \mathbb{R}$. **Hint:** Show that $f : S \to \mathbb{R} \times \mathbb{R}$ defined by $f \left( \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \right) = (a, c)$ is a homomorphism from $S$ onto $\mathbb{R} \times \mathbb{R}$ with kernel $I$. Now invoke the first isomorphism theorem.

**Solution:**

3. Let $p$ be a prime integer and let $T$ be the set of rational numbers (in lowest terms) whose denominators are not divisible by $p$. Let $I$ be the set of elements in $T$ such that the numerator is divisible by $p$.

   a. Prove $T$ is a ring and that $I$ is an ideal of $T$.

**Solution:**

   b. Show that $T/I \cong \mathbb{Z}_p$. **Hint:** See the hint for problem 2.

**Solution:**

**Extra Credit Problem**

4. Find the lattice of ideals of the ring $T$ from problem 3. There are infinitely many ideals but the lattice is pretty simple.

**Solution:**

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