MATH 412 HW 7

BILLY BOB

1. Let I be an ideal of a ring R and let S be a subring of R. Show that $I \cap S$ is an ideal of S.

Solution:

2. Let S be the set of matrices of the form $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ and let I be the set of matrices of the form $\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$, where a, b and c are in $\mathbb{R}$.

a. Show that S is a ring and that I is an ideal of S.

Solution:

b. Show that $S/I \cong \mathbb{R} \times \mathbb{R}$. **Hint:** Show that $f : S \to \mathbb{R} \times \mathbb{R}$ defined by $f\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = (a, c)$ is a homomorphism from S onto $\mathbb{R} \times \mathbb{R}$ with kernel I. Now invoke the first isomorphism theorem.

Solution:

3. Let p be a prime integer and let T be the set of rational numbers (in lowest terms) whose denominators are not divisible by p. Let I be the set of elements in T such that the numerator is divisible by p.

a. Prove T is a ring and that I is an ideal of T.

Solution:

b. Show that T/I is isomorphic to $\mathbb{Z}_p$. **Hint:** See the hint for problem 2.

Solution:

Extra Credit Problem

4. Find the lattice of ideals of the ring T from problem 3. There are infinitely many ideals but the lattice is pretty simple.

Solution: