

## MATH 412 HW 8

BILLY BOB

1. Let  $a$  and  $b$  be elements of a group. Prove that  $|bab^{-1}| = |a|$ .

**Solution:**

2. Let

$$A = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

be elements of  $\text{GL}(2, \mathbb{R})$ . Find the order of  $A$  and  $B$  and  $AB$ .

**Solution:**

3. Let  $G$  be a group and suppose  $(ab)^2 = a^2b^2$  for all  $a$  and  $b \in G$ . Prove that  $G$  is abelian.

**Solution:**

4. If  $H$  is a subgroup of a group  $G$  and  $a \in G$ , show that  $a^{-1}Ha := \{a^{-1}ha : h \in H\}$  is a subgroup of  $G$ . The **normalizer** of  $H$  is defined by

$$N(H) = \{g \in G : g^{-1}Hg = H\}.$$

Show that  $N(H)$  is a subgroup of  $G$  and that  $H \triangleleft N(H)$ .

**Solution:**