1. (Chap 10, #7) Prove or disprove: If $H$ is a normal subgroups of $G$ such that $H$ and $G/H$ are both abelian, then $G$ is abelian.

2. (Chap 10, #12) Show that if $G$ has exactly one subgroup $H$ of order $k$, then $H \triangleleft G$.

(20) 3. (Chap 10, #14) The center of a group $G$ is

$$Z(G) = \{x \in G : xg = gx \text{ for all } g \in G\}$$

a. Calculate the center of $S_3$.

b. Calculate the center of $\text{GL}_2(\mathbb{R})$.

c. Show that the center of any group $G$ is a normal subgroup of $G$.

d. Show that if $G/Z(G)$ is cyclic then $G$ is abelian.

4. An isomorphism of $G$ onto itself is called an automorphism of $G$. Prove that the set of all automorphisms of $G$ forms a group, $\text{Aut}(G)$.

5. Let $g \in G$ and let $i_g : G \rightarrow G$ be defined by $i_g(x) = gxg^{-1}$. Show that $i_g$ is an automorphism. An automorphism of this form is called an inner automorphism. Show that the set of all inner automorphisms forms a group (denoted $\text{Inn}(G)$) and that $\text{Inn}(G) \triangleleft \text{Aut}(G)$.