Math 412 Final, Take Home Part

Due Dec 15, 2015

You can use our text but no other books and you cannot use the internet.

1. Let G be the group $\mathbb{Z}_4 \times \mathbb{Z}_2$. G has 8 subgroups.

a. List them.

Solution:

b. List the complements of $\langle(2,1)\rangle$; that is, find those subgroups K such that $\langle(2,1)\rangle \cap K = \{0\}$ and $\langle(2,1)\rangle + K = G$.

Solution:

c. List the complements of $\langle(2,0)\rangle$.

Solution:

2. Let G be a group with $|G| < 100$ and suppose G has elements of orders 10 and 25. What is $|G|$?

Solution:

3. Show that a group of order 33 has an element of order 3.

Solution:

4. Let G be an abelian group and let T be the elements of G of finite order. Show that T is a subgroup and that the quotient group G/T has no elements of finite order except the identity.

Solution:

5. Let G be a group with identity element e and let a, $b \in G$. Assume $a \neq e$ and $b \neq e$. Also suppose

$$a^5 = e \quad \text{and} \quad aba^{-1} = b^2.$$

Find the order of b. Hint: Start by evaluating $a^2ba^{-2} = aaba^{-1}a^{-1}$, a^3ba^{-3}, etc.

Solution: