

Math 412–413 Introduction to Abstract Algebra

Introduction to basic algebraic structures. Groups, finite groups, abelian groups, rings, integral domains, fields, factorization, polynomial rings, field extensions, quotient fields. Emphasis on writing instruction. (These topics are covered in the year sequence Math 412–413.) The material is abstract and rigorous and requires mathematical maturity.

Pre for 412: 311 or 321 or consent.

Pre for 413: 412 or consent.

Ring Theory. Definition and examples, polynomial rings, ideals and quotient rings, modules, direct products, euclidean domains, principal ideal domains, unique factorization domains.

Group Theory. Definition and examples, permutation groups, the Cayley representation theorem, normal subgroups, quotient groups, direct products, the Isomorphism Theorems, abelian groups and the classification of finite abelian groups, the Sylow theorems.

Field Theory. Definition and examples, characteristic, vector spaces, polynomials over a field, construction of finite fields, field extensions and Galois Theory.

Math 412–413 is taught in a writing–intensive format. It is crucial that the student’s ability to understand, work with, and write about abstract systems be developed in this course.

Clear, concise writing that captures the essence of an idea characterizes good mathematical writing. With a few exceptions, the class assignments will involve the writing of proofs, or the explanation of examples or counterexamples. (This includes both homework and exams.) These assignments are of course collected and graded. The student must learn \TeX and all assignments will be written using it. Assignments must be written using proper grammatical English, with an effort towards clarity and conciseness. Poorly written assignments will be returned (with comments) to the student, who must then rewrite and resubmit the work for grading.

Written work (including homework and take–home exams, but not in–class exams) will make up at least 60% of a student’s course grade. *Students must adequately complete all writing assignments to pass the course with a D grade or better. Students who do not complete all writing assignments will fail the course.*