

## Math 475 Exercises 4

Due: Apr 20, 2010

1. The procedure below takes an array of integers and determines if some elements occurs three (or more) times in the array. Which of the following big- $O$  estimates:  $O(\log n)$ ,  $O(n)$ ,  $O(n \log n)$ ,  $O(n^2)$ ,  $O(n^2 \log n)$ ,  $O(n^3)$ ,  $O(n^3 \log n)$ ,  $O(n^4)$ , and  $O(2^n)$  best describes the worst-case running time of the algorithm.

```
public boolean hasThreeEqual(int [] arr) {
    int n = arr.length;
    for (int i = 0; i < n; i++) {
        for (int j = i + 1; j < n; j++) {
            if (arr[i] == arr[j]) {
                for (int k = j + 1; k < n; k++) {
                    if (arr[j] == arr[k]) {
                        return true;
                    }
                }
            }
        }
    }
    return false;
}
```

2. Show  $B(n) \leq n!$ .
3. Let  $V$  be a vector space of dimension 4 over a finite field with  $q$  elements and let  $\mathbf{L} = \mathbf{Sub}(V)$  be the lattice of subspaces. Find  $n = |\mathbf{L}|$  and the number  $e_{\prec} = e_{\prec}(\mathbf{L})$  of covers in this lattices. If you express  $e_{\prec}$  as powers of  $n$  and take the limit as  $q$  goes to infinity, it has the form  $cn^r$  plus lower order terms. Find  $c$  and  $r$ .
4. An  $n \times n$  matrix is *doubly stochastic* if  $0 \leq a_{ij} \leq 1$  and each row sum and each column sum is 1. Prove that if  $A$  is doubly stochastic then it has a diagonal all of whose entries are nonzero.