MATH 100 SECTION 1 FIRST BLOCK: CORE MATERIAL

Note that Stein’s formulations of named theorems are not necessarily the same as the ones given in class.

BASIC CONCEPTS

1. Types of numbers: natural numbers, integers, prime numbers, composite numbers, numbers which are squares, rational numbers, irrational numbers, repeating decimals, nonrepeating decimals, Lagado numbers and primes, enumers (positive even integers) and eprimes (enumers which are not products of smaller enumers)

2. one integer $A$ divides another integer $B$ (equivalently $A$ is a divisor or a factor of $B$ or $B$ is a multiple of $A$)

3. the sieve of Eratosthenese

4. twin prime

5. $\pi(n)$ (= the number of primes which are less than or equal to $n$)

6. Stein’s prime manufacturing machine

BASIC FACTS

1. a natural number $N$ is prime if it is not divisible by any prime $P$ with $P^2 \leq N$.

2. the lemma and three theorems on pages 18–19 of Stein

3. the prime number theorem and its associated results: Let $D$ denote the number of digits in a natural number $N$. Then for large $N$ (i) $\pi(N) \approx N/(2.3D)$; (ii) the $N$th prime is roughly $2.3 ND$; and (iii) the gap between the $N$th and the $N + 1$st prime is roughly $2.3 D$.

4. The fundamental theorem of arithmetic (= the unique factorization theorem): Every composite number can be written as a product of primes in exactly one way (except for the order of the factors).

5. the Pythagorean theorem (page 58)

6. theorems 1, 3, 4, 5, 6 on pages 63–66 of Stein

7. the rationality or irrationality of sums and products of rational and irrational numbers

SOME BASIC COMPUTATIONS

Finding the decimal form of a rational number and writing a repeating decimal as a quotient of integers.