

16.4 Cardinality

Start simple: How many elements are there in the following sets?

1. $\{\}$
2. $\{2, 4, 6, 4, 2\}$
3. $\{0, 1, 2\}$

What about \mathbb{N} ? \mathbb{R} ?

What if we don't know what "how many" means?

Start simpler: Define what it means for two sets to be the same size:

Definition: A set A is *conumerous* with B provided A and B can be put into a one-to-one correspondence. (Picture)

Other, equivalent notation/terminology for “ A is conumerous with B ”:

- a. A and B *have the same number of elements*
- b. A and B *have the same cardinality*
- c. $\text{card}(A) = \text{card}(B)$

Examples: 1. The sets above.
2. Finite sets and von Neumann natural numbers; definition of finite.

Galileo Galilei, 1638

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, \dots\} \quad (= \mathbb{N})$$

and

$$\{0, 1, 4, 9, 16, 25, 36, 49, 64, \dots\} \quad (= \{n^2 : n \in \mathbb{N}\})$$

are conumerous.

More examples:

1. \mathbb{N} and $\mathbb{N} - \{0\}$ (cf *Hilbert's Hotel*)
2. \mathbb{N} and \mathbb{Z}
3. Any two line segments (in the plane)
4. The real interval $(0, 1) \subset \mathbb{R}$ and all of \mathbb{R}