

## More examples

**I.** The algebraic numbers are countable.

**Proof** Stein.

Note that it follows that there exist at least one transcendental number.

**II.** Let  $S$  be the interior of the unit square in the plane,

$$S = \{(x, y) : 0 < x, y < 1\},$$

sometimes denoted by  $(0, 1) \times (0, 1)$ . Then  $S$  has cardinality  $\mathfrak{c}$ , that is,  $\text{card}(S) = \text{card}((0, 1))$ .

**Proof:**

**III. (Cantor)** For any set  $A$ ,  $\text{card}(A) \neq \text{card}(\mathcal{P}(A))$   
(in fact,  $\text{card}(A) < \text{card}(\mathcal{P}(A))$ )

Note that means that

$$\underset{\aleph_0}{\text{card}(\mathbb{N})} < \underset{2^{\aleph_0}}{\text{card}(\mathcal{P}(\mathbb{N}))} < \underset{2^{2^{\aleph_0}}}{\text{card}(\mathcal{P}(\mathcal{P}(\mathbb{N})))} < \dots$$

**GCH** (*General Continuum Hypothesis*) For any infinite set  $A$ , there is no cardinality between  $\text{card}(A)$  and  $\text{card}(\mathcal{P}(A))$

**Proof** of result III:

## 17 Probability and Statistics

### 17.1 Overview

The idea that probability is something we can study, analyze, and come to understand, is something relatively new:

*Fate laughs at probabilities.*

Bulwer Lytton

*How dare we speak of the laws of chance?*

*Is not chance the antithesis of all law?*

Joseph Bertrand, Calcul des probabilités

While basic probability theory grew out of investigations by gamblers in the late 18th century, a modern, rigorous foundation is fairly new:

*The theory of probability as a mathematical discipline can and should be developed from axioms in exactly the same way as geometry and algebra.*

Andrey Kolmogorov

*All possible definitions of probability fall short of the actual practice.*

William Feller

Besides helping us gamble if we are so inclined, understanding probability can help avoid jumping to conclusions:

*Coincidences, in general, are great stumbling blocks in the way of that class of thinkers who have been educated to know nothing of the theory of probabilities—that theory to which the most glorious objects of human research are indebted for the most glorious of illustrations.*

Edgar Allen Poe, *The Murders in the Rue Morgue*

*Lottery: A tax on people who are bad at math.*

Anonymous

*LAST NIGHT'S CHILLING LOTTERY  
WINNER: 9-1-1*

NY Post Headline, 12 September 2002

Statistics is historically less-respected than probability:

*There are three kinds of lies: lies, damned lies, and statistics.*

Benjamin Disraeli

*USA Today has come out with a new survey - apparently, three out of every four people make up 75% of the population.* David Letterman

*Smoking is one of the leading causes of statistics.*

Fletcher Knebel