Measures of spread (or dispersion):

- range=MAX-MIN
- inter-quartile range=IQR=$Q_3 - Q_1$
- sample variance
- sample standard deviation

$$s^2 = \text{sample variance} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$s = \text{sample standard deviation} = \sqrt{s^2}$$

Example: Data:

```
17  -1 -17 -25 -17 -15   1  -4   2  -8 -12
```

Find Min, Max, median, IQR, stem/leaf, mean, variance, standard deviation:

Probability space: $(\Omega, \mathcal{A}, P)$

Three basic elements: Sample Space Ω, Collection $\mathcal{A}$ of Events, Probability measure P

Sample Space: Ω =Sample Space=set of all possible "elementary outcomes"

Events: An *event* is a subset of Ω, so (usually) $\mathcal{A} = \mathcal{P}\ (\Omega))$

Probability: $P\colon \mathcal{A} \to [0,1]$ assigns values between 0 (completely improbable) and 1 (certain) to events.

Examples:

1. Roll one die, 'obvious' sample space is $\Omega = \{1, 2, 3, 4, 5, 6\}$

Take $\mathcal{A} = \mathcal{P}\ (\Omega)$.

Typical event: "Roll is even"$= \{2, 4, 6\}$

If die is 'fair', reasonable to take $P(A) = \frac{\#A}{6}$

$P(even) = P(\{2, 4, 6\}) = 3/6 = 1/2;$

$P(greater\ than\ 4) = P(\{5, 6\}) = 2/6 = 1/3$

If die is 'loaded', might have some numbers more likely than others.

One way to assign probabilities is to assign each number i a probability $p_i, 1 \leq i \leq 6$

Then let

$$P(A) = \sum_{i \in A} p_i$$

EG: If we have the following assignment:

1	2	3	4	5	6
.1	.2	.1	.1	.1	.4

then:

$P(even) = .2 + .1 + .4 = .7;$

$P(greater\ than\ 4) = .1 + .4 = .5;$

$P(\Omega) = .1 + .2 + .1 + .1 + .1 + .4 = 1$

2. Throw *two* dice; some possible sample spaces are:

$$\Omega_1 = \begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix}$$

or

$$\Omega_2 = \begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ & & (3,3) & (3,4) & (3,5) & (3,6) \\ & & & (4,4) & (4,5) & (4,6) \\ & & & & (5,5) & (5,6) \\ & & & & & (6,6) \end{matrix}$$

or

$$\Omega_3 = \{2, 3, 4, \ldots, 12\}$$

What are reasonable probability assignments for these different sample spaces?

Digression: ordered pairs, triples, n-tuples, Cartesian products

Definition: An *ordered pair* is an object (a, b) with the property:
$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$

EG: If $x, y \in \mathbb{R}$ then (x, y) is a point in the plane (and vice versa)

Note that order counts: if $(a, b) = (b, a)$ then $a = b$

So $(2, 3) \neq (3, 2)$ but $(3, 3) = (3, 3)$

Remark: Difference between $(2, 3)$ and $\{2, 3\}$

Can have ordered pairs of things other than real numbers, eg (George Bush, Aloha Tower)

Definition: If A and B are sets, the *Cartesian product* of A and B is

$$A \times B =_{def} \{(a, b) \colon a \in A, b \in B\}$$

EG: Famous Dogs × Transcendental Numbers =

$\{$(Lassie, π), (Rin Tin Tin, e), (Cujo, π), $\ldots\}$

More generally: Can generalize to *ordered triples* (a, b, c), or ordered $n-$tuples $(x_1, x_2, \ldots, x_n)$

EG: $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$ (or $\mathbb{R}^3$) = $\{(a, b, c) \colon a, b, c \in \mathbb{R}\}$ =3-dimensional space

The 4-tuple (Cujo, David Ross, Eiffel Tower, 17) is an element of
Famous Dogs × UH Math Professors × French Buildings × Primes

Counting principle: If A, B are finite sets, A has n elements, and B has m elements, then $A \times B$ has mn elements.

More generally, if $A_1, A_2, \ldots, A_k$ are finite with (respectively) $n_1, n_2, \ldots, n_k$ elements then $A_1 \times A_2 \times \cdots \times A_k$ is finite with $n_1 n_2 n_3 \cdots n_k$ elements.

EG: Roll 4 dice, number of different throws =?

EG: From a deck of cards, how many ways are there to pick one card from each suit?

3. Throw a dart at a dartboard of radius R.

Ω =all points in a circle (including interior) of radius R

An event will be any subset A of Ω for which "area of A" makes sense.

$P(A) =_{def}$(area of A)$/(\pi R^2)$

EG: P(hit upper half of dartboard) $= 1/2$

EG: If 'bull's-eye' of dartboard is circle of radius r, where $r < R$, then P(hit bull's-eye) $= \frac{\pi r^2}{\pi R^2} = (\frac{r}{R})^2$

Properties of a probability measure. In the definition of "probability space", we impose the following additional requirements on the probability function P:

$P(\emptyset) = 0$

$P(\Omega) = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (picture)

Definition Events A, B are *disjoint* if $A \cap B = \emptyset$

Note: If A and B are disjoint then

$$P(A \cup B) = P(A) + P(B)$$

EG: A pair of fair dice are thrown, what is $P($sum is odd or sum is 6$)$?