For a continuous random variable X with density function $f(x)$, the expected value is defined to be

$$E(X) =_{def} \int_{-\infty}^{\infty} x f(x) dx$$

For *any* random variable X, if $\mu = E(X)$, then the *Variance* of X is

$$Var(X) =_{def} E[(X - \mu)^2]$$

and the *Standard Deviation* of X is $\sqrt{Var(X)}$

If X discrete with distribution function $f(x)$, then $Var(X) = \sum_x (x - \mu)^2$

If X continuous with density function $f(x)$, then $Var(X) = \int_{-\infty}^{\infty} (x - \mu)^2 dx$

$E(X)$ and $Var(X)$ are *intrinsic properties* of the distribution of X

They are *not* empirical properties involving observed numbers (like the sample mean and variance).

EG: One can show (eg, in Math 373 or 471):

1. If X is Binomial(N, p) then $E(X) = np, Var(X) = npq$
2. If X is Normal with parameters μ, σ^2 then $E(X) = \mu, Var(X) = \sigma^2$

Properties: If X is a random variable with mean μ and variance σ^2 and $r \in \mathbb{R}$ then

$$E(rX) = rE(X); \qquad Var(rX) = r^2 Var(X)$$

$$\text{St'd Dev. of } rX = |r| \times \text{St'd Dev. of } X$$

Sums and samples: If $X_1, \ldots, X_n$ are independent random variables then

$$E(X_1+X_2+\cdots+X_n) = E(X_1)+E(X_2)+\cdots+E(X_n)$$

$$Var(X_1+X_2+\cdots+X_n) = Var(X_1)+Var(X_2)+\cdots+Var(X_n)$$

If $X_1, \ldots, X_n$ are IID random variables with mean μ and variance σ^2 then

$$E(X_1 + X_2 + \cdots + X_n) = n\mu$$

$$Var(X_1 + X_2 + \cdots + X_n) = n\sigma^2$$

$$E(\bar{X}) = \mu$$

$$Var(\bar{X}) = \frac{\sigma^2}{n}$$