**So:** The sample mean $\bar{X}$ is a pretty good **estimator** of the *actual* mean $\mu$.

In particular: $\bar{X}$ equals $\mu$ on average, and the variability shrinks as $n \to \infty$.

Similarly, the sample standard deviation is (usually) a pretty estimator of $\sigma$

(Technical term: these sample statistics are *consistent* estimators of $\mu, \sigma$.)

**CLT** Let $X_1, X_2, \ldots, X_n$ be a sample from a distribution with (actual) mean $E(X_i) = \mu$ and variance $\sigma^2$. Put

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Then (under reasonable hypotheses) for large $n, Z$ is approximately a "standard" normal, ie, density

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Probabilities of standard normals are tabulated in the front or back of most statistics textbooks. (examples)

Since $s$ is a good estimator of $\sigma$, we can usually replace $\sigma$ with $s$, and conclude that for large $n$,

$$Z = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

is approximately a standard normal.