11.3.1 Truth tables

Truth tables are useful tools to look at the behavior of a formula in all possible models. Each column is headed by a subformula of the formula being considered, with the simplest formulas (the proposition letters that appear in this formula). The rows contain all possible assignments of truth values to the proposition letters, and then the corresponding truth values of the subformulas are determined in increasing order of complexity.

If the column of truth values below the final formula consists entirely of **T**, then the formula is valid (or - in the book's terminology, a "tautology")

$$(A \vee (\sim A))$$

A	$(\sim A)$	$(A \vee (\sim A))$
T	F	T
F	T	T

(A tautology - the *law of excluded middle*)

The Basic Connectives

A	$(\sim A)$
T	F
F	T

A	B	$(A \Rightarrow B)$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	$(A \vee B)$
T	T	T
T	F	T
F	T	T
F	F	F

A	B	$(A \wedge B)$
T	T	T
T	F	F
F	T	F
F	F	F

Equivalence

A	B	$(\sim B)$	$(\sim A)$	$((\sim B) \Rightarrow (\sim A))$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Note that $(A \Rightarrow B)$ and its contrapositive

$$((\sim B) \Rightarrow (\sim A))$$

have the same truth values in all combinations of the proposition letters.

When two formulas have the same truth values - in other words, behave the same under all assignments of truth values - they are called *logically equivalent* (or just *equivalent*). The text uses the $\equiv$ symbol for this, for example,

$$(A \Rightarrow B) \equiv ((\sim B) \Rightarrow (\sim A)).$$

An implication is logically equivalent to its contrapositive.

Everyone should look at Section 3.1 of the text for a discussion of the relationship of the statement $A \Rightarrow B$, its contrapositive, and its *converse*

$$B \Rightarrow A$$

More Examples

(class)

1. Show $(\sim (A \vee B)) \equiv (\sim A \wedge \sim B)$
   (a "DeMorgan Law")

2. $A \Rightarrow (B \Rightarrow A)$

3. $(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$

4. $\sim\sim A \Rightarrow A$

5. Which of the following are valid? For those that are *not*, give a model in which they do not hold.

   (a) $(((A \Rightarrow B) \Rightarrow A) \Rightarrow A)$

   (b) $(((A \vee B) \wedge \sim A) \Rightarrow B)$

   (c) $(A \Rightarrow (\sim A \vee B))$

Question: If a formula has N proposition letters, how many rows will the truth table have?

Remark: More compact truth tables.