

Set Difference: The *difference* of two sets is the set of elements in one but not the other.

Equivalently,

$$A - B =_{\text{def}} \{x \mid x \in A \text{ and } x \notin B\}$$

Equivalently,

$$x \in (A - B) \text{ provided } (x \in A) \wedge (x \notin B)$$

Examples :

1. $\{1, 2, 3, 4\} - \{2, 4\} = ?$
2. $\{1, 2, 3, 4\} - \{2, 4, 6, 8\} = ?$
3. $\{2, 4\} - \{1, 2, 3, 4\} = ?$
4. $\{2, 4, 6, 8\} - \{1, 2, 3, 4\} = ?$
5. $\{1, 2, 3, 4\} - \{1, 2, 3, 4\} = ?$
6. In fact, for *any* set A , $A - A = \emptyset$
7. $\mathbb{N} - \{2k \mid k \in \mathbb{N}\} = ?$
8. $\{2k \mid k \in \mathbb{N}\} \cup \{2k + 1 \mid k \in \mathbb{N}\} = ?$

Complement: If $A \subset B$ then $B - A$ is the *complement* of A in B .

If all sets A under consideration are assumed to be subsets of some particular set X (*universe of discourse*) then $X - A$ is just the *complement* of A , and we denote it by A^c

Example The complement of the set of even natural numbers in \mathbb{N} is the set of odd natural numbers.

Pictures:

14.3 Some useful properties

Associative laws:

$$(1) A \cap (B \cap C) = (A \cap B) \cap C$$

$$(2) A \cup (B \cup C) = (A \cup B) \cup C$$

Distributive laws:

$$(3) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$(4) A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

DeMorgan laws:

$$(1) A - (B \cup C) = (A - B) \cap (A - C)$$

$$(1') (B \cup C)^{\complement} = B^{\complement} \cap C^{\complement}$$

$$(2) A - (B \cap C) = (A - B) \cup (A - C)$$

$$(2') (B \cap C)^{\complement} = B^{\complement} \cup C^{\complement}$$

Misc.

(1) If $A \subset B$ then $A \cap B = A$ and $A \cup B = B$

(2) (In fact, if $A \cap B = A$ or $A \cup B = B$ then $A \subset B$)

(For the rest of the properties, assume there's a universal set X)

(3) $(A^c)^c = A$

(4) $A \cap A^c = \emptyset$; $A \cup A^c = X$

(5) If $A \subset B$ then $B^c \subset A^c$