9.3 Conjunction, disjunction

Suppose A and B are propositions. The following are all different ways of saying the same thing:

$$A \text{ and } B \text{ are true.}$$
$$\text{Both } A \text{ and } B \text{ are true.}$$
$$A \text{ is true and } B \text{ is true.}$$
$$A \text{ and } B.$$
$$A \wedge B.$$
$$A \& B.$$

$A \wedge B$ is true when both A and B are true, and false if *either* of them is false. This connective is called *conjunction*.

Examples

1. $1 + 1 = 2$ and $2 + 2 = 4$
2. $1 + 1 = 2$ and $2 + 2 = 6$
3. Abraham Lincoln was the President of the United States and a famous cosmonaut.

Suppose A and B are propositions. The following are all different ways of saying the same thing:

A or B are true.
Either A or B are true.
Either A is true or B is true or both are true.
A is true or B is true.
A or B.
$A \vee B$.

$A \vee B$ is true when A is true or B is true *or both*, and false if *both* of them is *false*. This connective is called *disjunction*.

Examples

1. $1 + 1 = 2$ or $2 + 2 = 4$
2. $1 + 1 = 2$ or $2 + 2 = 6$
3. $1 + 1 = 3$ or $2 + 2 = 6$
4. Abraham Lincoln was the President of the United States or a famous cosmonaut.

10 Formal/Symbolic Logic - General

Some reasons to formalize logic:

- Make determining the truth of difficult logical statements a matter of simple calculation.
- Clarify which deductive methods (classical forms) are sound (correct) or necessary.
- Make it possible to formalize other disciplines (mathematics, computer science, others)
  - Automated theorem proving and program verification
  - Generalization through abstraction
  - Limits of mechanical thought; free will vs. determinism
- Problems with informal reasoning. (Ionesco, *Rhinosceros*; Schulman, *Life and loves of Dobie Gillis*)

Considerations in establishing a formal system:

- Should not be unnecessary cumbersome, difficult to use, or difficult to typeset
  (Examples: Lull et al; Lewis Carroll; Frege; Principia Mathematica; RPN; JSL)
- Should correspond to intuition.
- Should be extensive enough to cover all sitations of interest.
- Should be small enough to be tractable.
- (Propositional logic; Predicate Logic [aka First Order Logic]; Higher order logics and infinitary logics.)
- Should allow for a distinction between *syntactic* and *semantic* argument. (more later)
- Should be amenable to *metamathematical analysis*, for example, mathematical proofs about statements about the logic.