A brief history of solving polynomial equations

much stolen from:
http://www-history.mcs.st-andrews.ac.uk/history/index.html

Quadratic equation:

$$a_2x^2 + a_1x + a_0 = 0$$

400BC Babylonians could solve some problems that *we* would formulate as quadratic equations.

300BC Euclid: geometric solutions of some problems involving roots, that (again) *we* would formulate as quadratic equations.

628 Brahmagupta (in *Brahmasphutasiddhanta*, or The Opening of the Universe): solves relatively general quadratic equations

830 Abu Ja'far Muhammad ibn Musa al-Khwarizmi (member of the Banu Musa, scholars at the House of Wisdom in Baghdad), in *Hisab al-jabr w'al-muqabala*: specific (numerical) examples of several categories of quadratic equations, using geometric and algebraic methods (though all in words).

> *That fondness for science,…that affability and condescension which God shows to the learned, that promptitude with which he protects and supports them in the elucidation of obscurities and in the removal of difficulties, has encouraged me to compose a short work on calculating by al-jabr and al-muqabala , confining it to what is easiest and most useful in arithmetic.*

[al-jabr means "restoring", referring to the process of moving a subtracted quantity to the other side of an equation; al-muqabala is "comparing" and refers to subtracting equal quantities from both sides of an equation.]

113?CE Abraham bar Hiyya Ha-Nasi (aka Savasorda), in Hibbur ha-Meshihah ve-ha-Tishboret (Treatise on Measurement and Calculation); first published complete solution of the quadratic.

Cubic equations:

$$a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \quad (a_3 \neq 0)$$

Quartic equations:

$$a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \quad (a_4 \neq 0)$$

(c.f. www.sosmath.com/algebra/factor/fac111/fac111.html to see what cubic solution looks like)

1494 Fra Luca Pacioli *Summa de arithmetica, geometrica, proportioni et proportionalita*, shows the *quartic* $x^4 = a + bx^2$ can be solved by quadratic methods...

...but asserts $x^4 + ax^2 = b and x^4 + a = bx^2$ are impossible (why?), also says that the *cubic* cannot be solved in general.

1515 Scipione dal Ferro, University of Bologna, solves equations of form $x^3 + mx = n$, but keeps solution secret.

(This can be used to solve *all* cubics if you are comfortable with manipulating negative numbers.)

1526 dal Ferro dies, reveals solution on deathbed to his student Antonio Fior, who is evidently a braggart.

1535 Nicolo of Brescia, known as Tartaglia, thus learns that dal Ferro had the solution, and figures out what it must be. He then announces that he too can solve the cubic (without revealing his solution). Fior challenges him to a competition, each posing 30 problems to the other. Fior cannot solve any of the problems Tartaglia sets, but Tartaglia figures out a generalization of the basic solution, and solves all of Fior's problems instantly.

1539 Girolamo Cardano (illegitimate son of a lawyer/geometer who was a friend of da Vinci, and himself a doctor/mathematician) tries to get the solution from Tartaglia for a book in progress. He agrees he will only publish the method after Targaglia has a chance to publish it first, and swears:

> *I swear to you, by God's holy Gospels, and as a true man of honour, not only never to publish your discoveries, if you teach me them, but I also promise you, and I pledge my faith as a true Christian, to note them down in code, so that after my death no one will be able to understand them.*

1545 Cardano published *Ars Magna*, including solutions of both cubics and quartics (the latter mainly due to his student Lodovico Ferrari).

1673 Gottfried Wilhelm von Leibniz gives the easiest possible proof that the Cartan solutions actually work. (Class: show idea for quadratic.)

Casus Irreducibilis: Fior, Tartaglia, especially Cartan all noticed: even if a cubic has all three real roots, to solve for them algebraically you must take roots of nonreal numbers numbers at some point. Cartan called this the *casus irreducibilis*. Modern methods show that this is unavoidable.

Quintic: The 5th degree equation

$$a_5x^5+a_4x^4+a_3x^3+a_2x^2+a_1x+a_0 = 0, \qquad a_5 \neq 0$$

definitely has a solution:

1799 Paolo Ruffini announces the general quintic *cannot* be solved algebraically. Proof contains gaps, and announcement is largely ignored, but estabishes many mathematical facts that are later used.

1824 Niels Henrik Abel produces first correct proof of this unsolvability. Not published until well after his death.

1830++ Evariste Galois establishes a general algebraic theory which not only includes Abel's proof, but also a general framework for determining whether a given equation has an algebraic solution. His various papers on the subject get lost, rejected for spurious reasons, etc.

1831 French revolution. Galois arrested, tried, acquitted. Arrested again on Bastille day, while in prison learns his most important paper rejected for being badly written.

1832 Galois falls in love with daughter of prison physician. After release from prison tries to pursue her, is apparently rebuffed. Fights duel, possibly connected with her, is killed.

1843 French Academy of Science finally acknowledges Galois work as important, his papers get published in 1848.

Galois Theory and Impossibility:

Impossibility of solving the quintic.

Impossibility of trisecting an angle.