Sol'ns to Stein Ch. 2

4) No. N^2-1 factors: $N^2-1=(N-1)(N+1)$, and if $N\geq 3$ then both these factors are ≥ 2, so N^2-1 composite

8) A natural conjecture is "yes: $\forall N\in\mathbb{Z}^+$ $\exists p$ prime such that $N^2\leq p\leq (N+1)^2$" - however, this is still an open (= unsolved) question

27) To show: The last digit of any even number is 0, 2, 4, 6, or 8

Proof: First, recall (definition) that N even $\Rightarrow N=2K$ for some K, and $K=A+d$, where A ends in a 0 and d is the last digit of K, $0\leq d<9$. $2K=2(A+d)=2A+2d$. Since $2d$ ends in a 0, 2, 4, 6, 8 (since $2d$ is one of 0, 2, 4, 6, 8, 10, 12, 14, 16, 18), and $2A$ ends in a 0, $2A+2d$ ends in a 0, 2, 4, 6, or 8. Of course, $N=2A+2d$, so we're done.

32a) Given: Every even # >2 is sum of 2 primes

To Show: Every odd # >5 is sum of 3 primes.

Prf Let N odd, $N>5$. Then $N-3$ is an even number >2, so $N-3=p_1+p_2$ for some primes p_1 & p_2. Then $N=3+p_1+p_2$, which shows N is sum of 3 primes.

58) a) 4 (b) 8 (can you see why?)

57) Works for all 4 numbers.

Stein Ch. 3 sol'ns

16) a) Divisors of 96: $\{1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 36, 48, 96\}$

Divisors of 144: $\{1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144\}$

b) $$\left.\begin{aligned} 144 &= 96(1) + 48 \\ 96 &= \underbrace{48}(2) + 0 \end{aligned}\right\} \therefore GCD(96,144) = 48$$

c) $96 = 2^5 \times 3$, $144 = 2^4 \times 3^2$ $\therefore GCD = 2^4 \times 3 = 48$ $(LCM = 2^5 \times 3^2 = 288)$

40) Let $D = (A, BC)$. To show: $D = 1$. Else (for a contradiction) $p \mid D$ for some prime. Then $p \mid BC$ (since $p \mid D$ and $D \mid BC$), and $p \mid A$. Then $p \mid B$ or $p \mid C$; WOLG $p \mid B$. Then $p \mid A, p \mid B$ $\therefore p \mid GCD(A,B)$; but $GCD(A,B) = 1$ by hypothesis, contradiction.