# Extra Math 100 HW on Infinite Sets

1. Sketch a diagram illustrating why two circles of different diameters are conumerous.

2. Group the following sets by cardinality. (That is, which ones have the same cardinality as which others?) (a) $\mathbb{N}$ (b) $\mathbb{N} \times \mathbb{N}$ (c) $\mathbb{Q}$ (d) $\mathbb{R}$ (e) $\mathcal{P}(\mathbb{N}))$ (f) $[-1, 1]$ (g) $\mathcal{P}(\mathbb{R})$ (h) $\mathbb{R} \times \mathbb{R}$

3. What is the cardinality of the set of all English sentences? Why?

4. Let $E = \{A \subseteq \mathbb{N} \colon A \text{ is finite}\}$ (the set of all finite subsets of $\mathbb{N}$).
   (a) List some elements of $E$
   (b) List an element of $\mathcal{P}(\mathbb{N})$ which is *not* an element of $E$
   (c) If $A \in E$, let $x_A = 0.d_0 d_1 d_2 d_3 \ldots$ be the real number such that the $n^{th}$ digit $d_n = 1$ if $n \in A$, $d_n = 0$ if $n \in A^{\complement}$. Explain why this is a one-to-one correspondence of $E$ with a subset of the rational numbers. What can you conclude?

5. Is the Continuum Hypothesis true? False? Something else? (If so, what?)

6. Do $(0, 1)$ and $[0, 1)$ (subsets of $\mathbb{R}$) have the same cardinality? Explain.

7. In the proof of one theorem in class, we showed that if you have any sequence of real numbers, you can construct some new real number not in that sequence. What theorem was that?

8. What does the uncountability of $\mathbb{R}$ tell us about algebraic and transcendental numbers?