

# Answers to Extra Math 100 HW on Prob/Stat

## Part A

1a)  $\sum_{k=1}^6 k = 1 + 2 + 3 + 4 + 5 + 6 = 21$

1b)  $\sum_{k=2}^5 \sqrt{k-1} = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4}$

1c)  $\sum_{k=5}^8 k^2 - (k-1)^2 = 8^2 - 4^2$ . (More generally,  $\sum_{k=r}^s k^2 - (k-1)^2 = s^2 - (r-1)^2$ )

2) Stem/Leaf:

```
6 | 5
5 |
4 | 022
3 | 1289
2 | 3568999
```

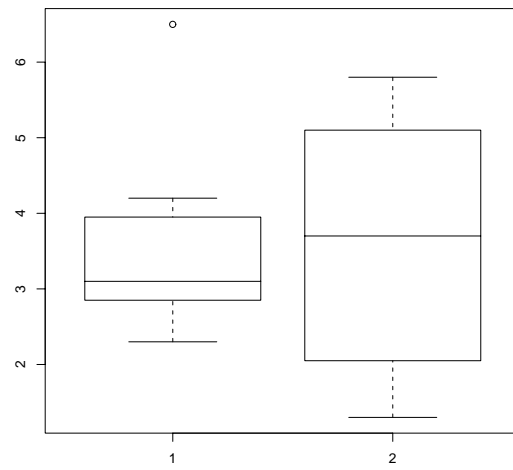
N=15 median=3.1 Q1=2.85 Q3=3.95 IQR=3.95-2.85=1.1 min=2.3 max=6.5  
boxplot see below

3) Stem/leaf:

```
5 | 026688
4 | 599
3 | 068
2 | 0166
1 | 3889
```

N=20 median=3.7 Q1=2.05 Q3=5.1 IQR=5.1-2.05=3.05 min=1.3 max=5.8

Boxplots for Problems 2 (left) and 3 (right):



4)  $\bar{x} = 2, s^2 = 20$ :

**Part B**

- 1a) Mean =  $\frac{3}{4}$ , Variance =  $(-2 - .75)^2/4 + (1 - .75)^2/2 + (3 - .75)^2/4 = 3.1875$
- 1b)  $A = 1 - (.1 + .1 + .2 + .2 + .1) = .3$ , Mean =  $0 * .1 + 1 * .1 + 2 * .2 + 3 * .2 + 4 * .1 + 5 * .3 = 3$ , Variance =  $(0 - 3)^2 * .1 + (1 - 3)^2 * .1 + (2 - 3)^2 * .2 + (3 - 3)^2 * .2 + (4 - 3)^2 * .1 + (5 - 3)^2 * .3 = 2.8$
- 2)  $E(X) = 0.25 \times 72 = 18$
- 3) If  $Y$  = winnings, then  $Y = 5X + (-1)(72 - X)$  (why?), so  $E(Y) = E(5X - 72 + X) = 5E(X) - 72 + E(X) = 5 * 18 - 72 + 18 = 36$  dollars.
- 4)  $E(X^3) = (\frac{1}{6})1^3 + (\frac{1}{6})2^3 + (\frac{1}{6})3^3 + (\frac{1}{6})4^3 + (\frac{1}{6})5^3 + (\frac{1}{6})6^3 = \frac{441}{6} = 73.5$
- 5) Probability of a 5 or 6 is  $\frac{1}{3}$ , so expected number of successes  $36/3 = 12$ .
- 6a)  $\sigma_X = 2, \sigma_Y = 1, \sigma_Z = 3$
- 6b)  $E(X + Y) = E(X) + E(Y) = 1 - 2 = -1$ ,  $E(5Y) = 5E(Y) = -10$ ,  $E(2X + Y + 2Z) = 2E(X) + E(Y) + 2E(Z) = 2 - 2 + 6 = 6$ , and  $E(X - Y) = E(X) - E(Y) = 1 - (-2) = 3$ .  $Var(X + Y) = Var(X) + Var(Y) = 5$ , Std.Dev =  $\sqrt{5}$ ,  $Var(5Y) = 25Var(Y) = 25$ , Std.Dev =  $\sqrt{25} = 5$ ,  $Var(2X + Y + 2Z) = 4Var(X) + Var(Y) + 4Var(Z) = 16 + 1 + 36 = 53$ , Std.Dev =  $\sqrt{53}$ , and  $Var(X - Y) = Var(X) + Var(Y) = 4 + 1 = 5$ , Std.Dev =  $\sqrt{5}$ .
- 7) NOTE TYPO; problem should have started: "In a class of 49 students, the heights  $x_1, x_2, \dots, x_{49}$ "  $\bar{x} = 68, s^2 = 4$ .
- 7a) 95% confidence interval for the height is  $\bar{x} \pm z_{[.025]} \frac{s}{\sqrt{49}} = 68 \pm (1.96) \frac{2}{7}$
- 7b) 99% confidence interval for the height is  $\bar{x} \pm z_{[.005]} \frac{s}{\sqrt{49}} = 68 \pm (2.58) \frac{2}{7}$
- 8) THIS PROBLEM IS AN ERROR - I gave no measured value, so you can't compute a confidence interval! Try again, with a measured value of 25 heads out of 72 flips.
- 9)  $1/2$