Table of Integrals (from H-H)

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\sin\alpha \sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$

$\sin\alpha \cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right)$

$\cos\alpha \cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$
2. $\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\int a^x\,dx = \frac{1}{\ln a}a^x + C, \quad a > 0$
4. $\int \ln x\,dx = x\ln x - x + C$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \tan x\,dx = -\ln|\cos x| + C$

II. Products of e^x, $\cos x$, and $\sin x$

8. $\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\sin(bx) - b\cos(bx)] + C$
9. $\int e^{ax}\cos(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\cos(bx) + b\sin(bx)] + C$
10. $\int \sin(ax)\sin(bx)\,dx = \frac{1}{b^2-a^2}[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)] + C, \quad a \neq b$
11. $\int \cos(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)] + C, \quad a \neq b$
12. $\int \sin(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x, $\cos x$, $\sin x$

13. $\int x^n \ln x\,dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1$
14. $\int p(x)e^{ax}\,dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a}\int p'(x)e^{ax}\,dx$
$$= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \cdots$$
$(+ - + - \ldots)$ (signs alternate)
15. $\int p(x)\sin ax\,dx = -\frac{1}{a}p(x)\cos ax + \frac{1}{a}\int p'(x)\cos ax\,dx$
$$= -\frac{1}{a}p(x)\cos ax + \frac{1}{a^2}p'(x)\sin ax + \frac{1}{a^3}p''(x)\cos ax - \cdots$$
$(- + + - - + + \ldots)$ (signs alternate in pairs after first term)
16. $\int p(x)\cos ax\,dx = \frac{1}{a}p(x)\sin ax - \frac{1}{a}\int p'(x)\sin ax\,dx$
$$= \frac{1}{a}p(x)\sin ax + \frac{1}{a^2}p'(x)\cos ax - \frac{1}{a^3}p''(x)\sin ax - \cdots$$
$(+ + - - + + - - \ldots)$ (signs alternate in pairs)

IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx, \quad n$ positive
18. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx, \quad n$ positive
19. $\int \frac{1}{\sin^m x}\,dx = \frac{-1}{m-1}\frac{\cos x}{\sin^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2}x}\,dx, \quad m \neq 1, m$ positive
20. $\int \frac{1}{\sin x}\,dx = \frac{1}{2}\ln\left|\frac{(\cos x) - 1}{(\cos x) + 1}\right| + C$ **OR:** $\int \csc x\,dx = \ln|\csc x - \cot x| + C$
21. $\int \frac{1}{\cos^m x}\,dx = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2}x}\,dx, \quad m \neq 1, m$ positive
22. $\int \frac{1}{\cos x}\,dx = \frac{1}{2}\ln\left|\frac{(\sin x) + 1}{(\sin x) - 1}\right| + C$ **OR:** $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
23. $\int \sin^m x\cos^n x\,dx$: If m is odd, let $w = \cos x$. If n is odd, let $w = \sin x$. If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. If both m and n are even and negative, the substitution $w = \cos x$ converts the integral into a rational function which can be integrated by the method of partial fractions.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$
25. $\int \frac{bx+c}{x^2+a^2}\,dx = \frac{b}{2}\ln|x^2+a^2| + \frac{c}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$
26. $\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}(\ln|x-a| - \ln|x-b|) + C, \quad a \neq b$
27. $\int \frac{cx+d}{(x-a)(x-b)}\,dx = \frac{1}{a-b}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + C, \quad a \neq b$

VI. Integrands Involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $\quad a > 0$

28. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + C$
29. $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln|x + \sqrt{x^2 \pm a^2}| + C$
30. $\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\,dx\right) + C$
31. $\int \sqrt{x^2 - a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2 - a^2} - a^2\int \frac{1}{\sqrt{x^2 - a^2}}\,dx\right) + C$