Derivatives

Theorem 0.1. *(Derivatives of basic functions)*

1. *If* $f(x) = k$ *where* k *is a constant, then* $f'(x) = 0$

$$\frac{d}{dx}k = 0$$

2. *If* $f(x) = x^n$ *where* $n > 0$ *is a positive integer, then* $f'(x) = nx^{n-1}$

$$\frac{d}{dx}x^n = nx^{n-1}$$

3. *If* $f(x) = \sqrt{x}$ *and* $x > 0$ *then* $f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

4. *If* $f(x) = \sin x$ *then* $f'(x) = \cos x$

$$\frac{d}{dx}\sin x = \cos x$$

5. *If* $f(x) = \cos x$ *then* $f'(x) = \sin x$

$$\frac{d}{dx}\cos x = -\sin x$$

Proof. Class. □

Theorem 0.2. *(Construction rules)*

Assume f and g are differentiable at x, and k is a constant.

1. $(kf)'(x) = kf'(x)$

$$\frac{d}{dx}kf(x) = k\frac{d}{dx}f(x)$$

2. $(f+g)'(x) = f'(x) + g'(x)$

$$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

3. $(f-g)'(x) = f'(x) - g'(x)$

$$\frac{d}{dx}(f(x)-g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

4. $(fg)'(x) = f(x)g'(x) + f'(x)g(x)$

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

5. $(\frac{f}{g})'(x) = \frac{g(x)f'(x)-f(x)g'(x)}{g(x)^2}$

$$\frac{d}{dx}(\frac{f(x)}{g(x)}) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{g(x)^2}$$

Proof. Class. □

Example 0.1. *(Applications: derivatives of some more functions)*

1. *Polynomials:* $\frac{d}{dx}(a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_nx^n) =?$

2. $\frac{d}{dx}x^n =?$ *when* $n < 0$ *is an integer and* $x \neq 0$

3. *Other trig functions:* $\frac{d}{dx}\tan x =?$

   $\frac{d}{dx}\sec x =?$

   $\frac{d}{dx}\cot x =?$