

In Exercises 1–40, evaluate the given integral.

1. $\int \frac{x^2}{x^2 + 2x - 15} dx$
3. $\int \frac{1}{1 - \sin \theta} d\theta$
5. $\int e^{2x} \sin 3x dx$
7. $\int \frac{\ln(2x)}{x^2} dx$
9. $\int \sqrt{4 - x^2} dx$
11. $\int \frac{-12}{x^2\sqrt{4 - x^2}} dx$
13. $\int \sec^4 \frac{x}{2} dx$
15. $\int \frac{9}{x^2 - 9} dx$
16. $\int \frac{\sec^2 \theta}{\tan \theta (\tan \theta - 1)} d\theta$
17. $\int \frac{x^2 + 2x}{x^3 - x^2 + x - 1} dx$
19. $\int \frac{3x^3 + 4x}{(x^2 + 1)^2} dx$
21. $\int \frac{16}{\sqrt{16 - x^2}} dx$
23. $\int \frac{e^x}{4 + e^{2x}} dx$
25. $\int \frac{x}{x^2 + 4x + 8} dx$
27. $\int \theta \sin \theta \cos \theta d\theta$
29. $\int (\sin \theta + \cos \theta)^2 d\theta$
30. $\int \cos 2\theta (\sin \theta + \cos \theta)^2 d\theta$
31. $\int \frac{x^{1/4}}{1 + x^{1/2}} dx$
33. $\int \sqrt{1 + \cos x} dx$
35. $\int \ln(x^2 + x) dx$
37. $\int \cos x \ln(\sin x) dx$
39. $\int \frac{x^4 + 2x^2 + x + 1}{(x^2 + 1)^2} dx$
40. $\int \sqrt{1 + \sqrt{x}} dx$
2. $\int \frac{\sqrt{x^2 - 9}}{x} dx$
4. $\int x^2 \sin 2x dx$
6. $\int (x^2 - 1)e^x dx$
8. $\int 2x\sqrt{2x - 3} dx$
10. $\int \frac{\sqrt{4 - x^2}}{2x} dx$
12. $\int \tan \theta \sec^4 \theta d\theta$
14. $\int \sec \theta \cos 2\theta d\theta$
18. $\int \frac{4x - 2}{3(x - 1)^2} dx$
20. $\int \sqrt{\frac{x - 2}{x + 2}} dx$
22. $\int \frac{\sin \theta}{1 + 2 \cos^2 \theta} d\theta$
24. $\int \frac{x}{x^2 - 4x + 8} dx$
26. $\int \frac{3}{2x\sqrt{9x^2 - 1}} dx$
28. $\int \frac{\csc \sqrt{2x}}{\sqrt{x}} dx$
32. $\int \sqrt{1 - \cos x} dx$
34. $\int \ln \sqrt{x^2 - 1} dx$
36. $\int x \arcsin 2x dx$
38. $\int e^x \arctan e^x dx$

In Exercises 41–44, evaluate the given integral the indicated methods.

41. $\int \frac{1}{x^2\sqrt{4 + x^2}} dx$
 - (a) trigonometric substitution
 - (b) substitution: $x = 2/u$
42. $\int \frac{1}{x\sqrt{4 + x^2}} dx$
 - (a) trigonometric substitution
 - (b) substitution: $u^2 = 4 + x^2$
43. $\int \frac{x^3}{\sqrt{4 + x^2}} dx$
 - (a) trigonometric substitution
 - (b) substitution: $u^2 = 4 + x^2$
 - (c) by parts: $dv = (x/\sqrt{4 + x^2}) dx$
44. $\int x\sqrt{4 + x} dx$
 - (a) trigonometric substitution
 - (b) substitution: $u^2 = 4 + x$
 - (c) substitution: $u = 4 + x$
 - (d) by parts: $dv = \sqrt{4 + x} dx$

52. Let

$$I_n = \int_0^{\infty} \frac{x^{2n-1}}{(x^2 + 1)^{n+3}} dx, \quad n \geq 1.$$

Prove that

$$I_n = \left(\frac{n-1}{n+2}\right) I_{n-1}$$

and then evaluate the following.

(a) $\int_0^{\infty} \frac{x}{(x^2 + 1)^4} dx$

(b) $\int_0^{\infty} \frac{x^3}{(x^2 + 1)^5} dx$

(c) $\int_0^{\infty} \frac{x^5}{(x^2 + 1)^6} dx$

53. Verify the reduction formula

$$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx.$$

54. Show that

$$\int_x^1 \frac{1}{1+t^2} dt = \int_1^{1/x} \frac{1}{1+t^2} dt$$

by evaluating each integral and then using the identity

$$\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}, \quad x > 0.$$