Exercises 4

1. (a) Suppose that $\mathcal{F}$ is a family of open disks in the plane, no two of which intersect. Show that $\mathcal{F}$ must be countable. (Hint: if you have trouble with that, first try the following simpler problem: Suppose that $\mathcal{G}$ is a family of *disjoint* open intervals in the line. Show that $\mathcal{G}$ must be countable. The proof of this is exactly the same as the proof that $\mathcal{F}$ must be countable.)
   (b) Suppose $\mathcal{H}$ is a family of *circles* in the plane; need $\mathcal{H}$ be countable? If not, why not?

2. Show that the set of real numbers with eventually-repeating decimal expansions is countable.

3. Let E be a set of positive real numbers with the property that for every finite subset $E' \subseteq E$, $\sum_{e \in E'} e < 17$. Show that E must be countable.

4. Prove that the set of all lines in the plane has cardinality $2^{\aleph_0}$.

5. Show that if A is a finite set with cardinality n and $B \subseteq A$ then B is a finite set with some cardinality $m \leq n$. (Hint: induct on n.)