Exercises on Ordinals

1. Let $(A, \prec)$ be a well-ordered set, and B any subset of A. Of course, B is also well-ordered by $\prec$. If α is the order type of $(A, \prec)$, and β the order type of $(B, \prec)$, show that $\beta \in \alpha$ or $\beta = \alpha$

2. Suppose that α and β are ordinal numbers with $\alpha \in \beta$. Show that $\alpha^+ \in \beta^+$. Conclude that for any ordinals $\alpha \neq \beta$, $\alpha^+ \neq \beta^+$ (ie, the "+" operation is injective on the class of ordinals).

3. Suppose that E is a set, that R is a relation on E, and that both R and R^{-1} are well-orderings on E. Show that E must be finite.

4. Let ω_1 be the smallest uncountable ordinal number. Suppose $A \subset \omega_1$ and A is countable. Show that there is an ordinal $\beta < \omega_1$ such that $A \subseteq \beta$.