

Exercises 7(?)

1. Let \mathcal{L} be the language $\mathcal{L} = \langle c, f, P \rangle$, where c is a constant symbol, f is a unary function symbol, and P is a binary relation symbol. For each of the following formulas ϕ , list the free variables of ϕ , and state whether or not ϕ is a *sentence* of \mathcal{L} .

- (a) $\forall x \exists y P(y, x)$
- (b) $c = f(f(x))$
- (c) $\forall x (P(f(x), c) \wedge P(f(x), c))$
- (d) $\exists x (x = f(x))$
- (e) $\forall x P(x, f(x))$

2. A model \mathfrak{A} for the language \mathcal{L} (from problem 1) is defined by

$$\mathfrak{A} = \langle \mathbb{N}, 0_{\mathbb{N}}, f_{\mathfrak{A}}, <_{\mathbb{N}} \rangle,$$

where $f_{\mathfrak{A}}(x) = x + 1$ for all $x \in \mathbb{N}$. For each of the formulas in 1(a)-(e) above, if the formula is a sentence state whether or not it holds in \mathfrak{A} .

3. A model \mathfrak{B} for the language \mathcal{L} (from problem 1) is defined by

$$\mathfrak{B} = \langle \mathbb{Q}, 0_{\mathbb{Q}}, f_{\mathfrak{B}}, <_{\mathbb{Q}} \rangle,$$

where $f_{\mathfrak{B}}(x) = x/2$ for all $x \in \mathbb{Q}$. For each of the formulas in 1(a)-(e) above, if the formula is a sentence state whether or not it holds in \mathfrak{B} .

4. A model \mathfrak{C} for the language \mathcal{L} (from problem 1) is defined by

$$\mathfrak{C} = \langle \mathbb{N}, 2_{\mathbb{Q}}, f_{\mathfrak{C}}, P_{\mathfrak{C}} \rangle,$$

where $f_{\mathfrak{C}}(x) = x + 1$ for all $x \in \mathbb{N}$ and $P_{\mathfrak{C}} = \{(a, b) : a \text{ evenly divides } b\}$. For each of the formulas in 1(a)-(e) above, if the formula is a sentence state whether or not it holds in \mathfrak{C} .