# Math 471 Homeworks – Fall 2004

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## 1 Assignment 1 - due 3 September 2004

1. Text problem 2.15

2. Text problem 2.19

3. A point is selected at random from unit disk $\{(x, y) : x^2 + y^2 \leq 1\}$. What is the probability that the point is at least 1/2 unit from the origin?

4. A box has 10 balls numbered 1 through 10. A ball is picked at random from this box, then another is selected at random from the remaining 9 balls. Find the probability that the numbers on the two balls differ by at least 2.

5. Two standard 6-sided dice are rolled; what is the probability that the resulting throw (ie, the sum of the spots on the uppermost sides) is an even number?

In all these problems, make sure you explicitly describe the sample space.

## 2 Assignment 2 - due 17 September 2004

1. Text problem 4.6

2. Let $\Omega = \mathbb{R}^2$, and $\mathcal{B}$ be the smallest $\sigma$-algebra containing all products of the form $I \times J$, where $I$ and $J$ are intervals (open or closed or half-open) in the real line. (a) Show that any horizontal or vertical line in $\Omega$ is in $\mathcal{B}$. (b) Is the line $\{(x, y) : y = x\}$ in $\mathcal{B}$? Why or why not?

## 3 Assignment 3 - due 27 September 2004

1. Text problem 6.19
2. Text problem 6.25
3. Text problem 6.33
4. Text problem 6.41

## 4 Assignment 5 - due 13 October 2004

1. Text problem 8.23
2. Suppose $X \sim Geo(p)$ (a) What is the pdf of $X^2$? (b) What is the pdf of $\max\{X, M\}$, where $M$ is a fixed positive number?
3. Suppose $X \sim Exp(\lambda)$; what is the pdf of $X^2$?

## 5 Assignment 6 - due 20 October 2004

1. Text problem 8.23
2. Suppose $X \sim Geo(p)$ (a) What is the pdf of $X^2$? (b) What is the pdf of $\max\{X, M\}$, where $M$ is a fixed positive number?
3. Suppose $X \sim Exp(\lambda)$; what is the pdf of $X^2$?

## 6 Assignment 7 - due 12 November 2004

1. Suppose $X \sim Geo(p)$, and $a, b$ are positive integers (a) Show that $P(X > a + b | X > a) = P(X > b)$ (b) What does this tell you about process described by a geometric random variable? (c) Suppose we know that $X = x$. For what value of $p$ is this most likely? In other words, if we view $P(X = x)$ as a function of $p$, find the value of $p$ which maximizes this.

2. Let $X \sim Binom(n, p)$. Find $E(X^3)$ by 'brute force' (that is, without using moment generating functions). Hint: Find $E(X(X-1)(X-2))$ first.

3. Let $(X, Y)$ be the coordinates of a point dropped "at random" inside the unit circle centered at the origin. That is, the joint pdf of $(X, Y)$ is $f(x, y) = (1/\pi)\mathbb{1}_{x^2+y^2 \leq 1}$. Find: (a) The marginal density of $X$; (b) $f(y|x)$; (c) $P(X \leq Y)$

4. The number $Y$ of defects per yard of a fabric is known to have a Poisson distribution woith parameter $\lambda$ However, $\lambda$ is not known, and in fact $\lambda$ is itself a random variable with the $EXP(1)$ distribution. What is the *unconditional* distribution of $Y$?

# 7 Assignment 8 - due 17 November 2004

1. If $X$ has moment generating function $M(t)$, and $a, b \in \mathbb{R}$, find the moment generating function of $aX + b$ in terms of that for $X$.

2. Let $X$ be uniformly distriubuted on $(a, b)$. Find the moment generating function for $X$, and use it to find the mean and variance of $X$.

3. Let $X$ have the bi-exponential distribution, with pdf $f(x) = \frac{1}{2}e^{-|x|}$, $x \in \mathbb{R}$. Find the moment generating function for $X$.

4. Problem #4 from the moment generating function handout (where $X$ and $Y$ are iid random variables with MGF $M(t) = \frac{\alpha}{\alpha - t}$, and you are to find the MGF for $X + Y$, $2X$, and draw conclusions about the pdfs).