5.5#4 $P_p(\text{reject } H_0) = P_p(X_1 \le 3) = \begin{cases} .17 & \text{if } p = 1/2 \\ .78 & \text{if } p = 1/4 \end{cases}$ (from Binom. tables)

$\therefore\ \alpha = P_{H_0}(\text{reject } H_0) = .17,$

Power $= 1-\beta = P_{H_a}(\text{reject } H_0) = .78$

5.5#1 $\eta(\theta) = P_\theta(\text{reject } H_0) = P_\theta(X_{(4)} \ge c) = 1 - P_\theta(X_{(4)} < c) = 1 - \left(\frac{c}{\theta}\right)^4 \equiv$ power fcn.

(since $X_i \sim \mathcal{U}(0,\theta)$). $\alpha = \eta(1) = 1 - \left(\frac{c}{1}\right)^4 = 1 - c^4$

(a) Set $\alpha = .05$, $(c/1)^4 = .95$ $\therefore\ c = \sqrt[4]{.95} \simeq .987$

5.6#5 Assume $X_i \underset{\text{iid}}{\sim} \mathcal{N}(\mu, \sigma^2)$, test stat. is $T = \frac{\bar{X} - 10.1}{s/\sqrt{n}} = \frac{\bar{X} - 10.1}{s/4}$,

reject if $T > t_{15,.05} = 1.753$. For given data, $T = \frac{10.4 - 10.1}{.4/4} = 3$

so, reject H_0; $p = P(T_{15} > 3) < .005$ from table.

5.6#8 $H_0: p = .14$ vs. $H_a: p > .14$. Let $Y = \#$ wearing belts $\sim Bin(n, p)$

$\hat{p} = \frac{Y}{n} \simeq \mathcal{N}(p, \frac{pq}{n})$, statistic $Z = \frac{\hat{p} - p_0}{\sqrt{pq/n}} \sim \mathcal{N}(0,1)$ (approx) under H_0

reject if $Z > z_\alpha = z_{.05}$ (say, for 95% conf level) $= 1.65$

In this case, $Z = \frac{\left(\frac{104}{590} - .14\right)}{\sqrt{(.14)(.86)/590}} = 2.539 > 1.65,$

so reject at 95% level,

$p = P(\mathcal{N}(0,1) > 2.539) = .0055$, so reject even at $\alpha = .01$ level.