6.3/8 8.1/8 8.2/5, 6 8.3 –

6.3#8 a) $L_\theta = \theta^{\sum x_i} e^{-\theta n}/(x_1 x_2 \cdots x_n)$, so $LL_\theta = n\bar{x}\ln\theta - \theta n - \ln(x_1\cdots x_n)$

$\frac{\partial LL}{\partial\theta} = \frac{n\bar{x}}{\theta} - n$ set $=0$, $\hat{\theta} = \bar{x}$; $\frac{\partial^2 LL}{\partial\theta^2} = \frac{-n\bar{x}}{\theta^2} - 1 < 0$, so $\hat{\theta} = \bar{x}$ is MLE

So $\Lambda = \frac{L_{\theta_0}}{L_{\hat{\theta}}} = \theta_0^{n\bar{x}} e^{-\theta_0 n} / \bar{x}^{n\bar{x}} e^{-n^2\bar{x}} = \theta_0^{nY} e^{-\theta_0 n} Y^{-nY} e^{n^2 Y} = \Lambda(Y)$;

LR test has form "reject if $\Lambda(Y) < K$", which is based on Y.

(In fact, $\Lambda(Y) < K$ is equiv. to $(\theta_0^n e^{n^2})^Y Y^{-nY} < K'$, equiv to

$(\theta_0 e^n)^Y Y^{-Y} < K''$, and if you draw the graph of

$w = (\frac{\gamma}{y})^y$ it increases in y to $\frac{\gamma}{e}$, then decreases,

so reject if Y large or small)

b) $P_{H_0}(\text{reject}) = P(Y \le 4 \text{ or } Y \ge 16 \mid Y \sim \text{Poiss}(10)) = 1 - \sum_{k=5}^{15} 10^k e^{-10}/k! \cong .078$

8.1 #8 Under H_0, $X_i \sim \mathcal{U}(0,1)$; under H_1, X_i has pdf $\frac{\Gamma(2+2)}{\Gamma(2)\Gamma(2)} x^{2-1}(1-x)^{2-1} = 3!\, x(1-x)$

so best test has form: reject if $\frac{L_1(\underset{\sim}{x})}{L_0(\underset{\sim}{x})} \ge k$, $(3!)^n (x_1\cdots x_n)(1-x_1)\cdots(1-x_n) \ge k$,
or $\prod_{i=1}^n x_i(1-x_i) \ge c$ for some c

8.2 #5 In the example, if $0 < \theta'' < \theta'$, then $\left(\frac{\theta''}{\theta'}\right)^{n/2} \exp\left[-\underbrace{\left(\frac{\theta''-\theta'}{2\theta'\theta''}\right)}_{<0} \sum x_i^2\right] \le k$

is best for $H_0: \theta = \theta'$ vs $H_a: \theta = \theta''$. Then $\theta''-\theta' < 0$, and $\left(\frac{\theta''}{\theta'}\right)$ constant,

so this is equivalent to "reject if $\sum x_i^2 \le c$" for some c.

Since this doesn't depend on the choice of $\theta'' < \theta'$, the test is UMP

for $H_0: \theta = \theta'$ vs $H_a: \theta < \theta'$.

8.2#6 Since "reject if $\sum x_i^2 \le c$" is UMP for $H_a: \theta < \theta'$,
and "reject if $\sum x_i^2 \ge c$" is UMP for $H_a: \theta > \theta'$,

and $\{\underset{\sim}{x} \mid \sum x_i^2 \le c\} \ne \{\underset{\sim}{x} \mid \sum x_i^2 \ge d\}$ for <u>any</u> c, d,

no test is UMP for $H_a: \theta \ne \theta'$.