7.1/4,6 7.2/6,7 7.3

7.1/ (4) Note $\theta = E(k_1 Y_1 + k_2 Y_2) = k_1 E(Y_1) + k_2 E(Y_2) = k_1\theta + k_2\theta$, so $k_1 + k_2 = 1$

Let σ^2 = Variance of Y_2, then $Var(Y_1) = 2\sigma^2$, and we want to minimize

$$w = Var(k_1 Y_1 + k_2 Y_2) = k_1^2(2\sigma^2) + k_2^2\sigma^2 = \left(2k_1^2 + (1-k_1)^2\right)\sigma^2$$

$$\frac{dw}{dk_1} = 4k_1 + 2(1-k_1)(-1) = 6k_1 - 2 \quad \text{set} = 0, \quad \boxed{k_1 = 1/3 \quad (\text{so } k_2 = 2/3)}$$

Check for minimality: $\frac{d^2w}{dk_1^2} = 6 > 0$, so this is a min.

(6) Note $E(X_i) = Var(X_i) = \theta$, so $E(Y) = n\theta$, $Var(Y) = n\theta$, $E(Y^2) = n\theta + (n\theta)^2$

$$R(\theta,\delta) = E((\delta-\theta)^2) = E(((b+\tfrac{Y}{n})-\theta)^2) = E((b+\tfrac{Y}{n})^2 - 2\theta(b+\tfrac{Y}{n}) + \theta^2)$$

$$= E\left(b^2 + \frac{2b}{n}Y + \frac{Y^2}{n^2} - 2\theta b - \frac{2\theta}{n}Y + \theta^2\right)$$

$$= b^2 + \left(\frac{2b}{n}\right)(n\theta) + \left(\frac{1}{n^2}\right)(n\theta + (n\theta)^2) - 2\theta b - \left(\frac{2\theta}{n}\right)(n\theta) + \theta^2$$

$$= (8^{th} \text{ grade algebra}) \; b^2 + \frac{\theta}{n}.$$

Note for any fixed θ, this is minimized when $b = 0$

$\therefore$ minimum-risk estimator of this form is $\delta = \frac{\theta}{n}$,

and $\max_\theta \frac{\theta}{n} = \infty$,

7.2 #6 pdf is $K_\theta x^\theta (1-x)^2$, so joint pdf is $\left[(K_\theta)^n (x_1 \cdots x_n)^\theta\right]\left[(1-x_1)(1-x_2)\cdots(1-x_n)\right]^2$,
so $(x_1 \cdots x_n)$ is sufficient for θ by Fisher/Neyman factorization

#7 pdf is $K_\theta x^{\theta-1} e^{-x/6}$, so joint pdf is $\left[(K_\theta)^n (x_1 \cdots x_n)^{\theta-1}\right] e^{-\frac{1}{6}\sum x_i}$
and $(x_1 \cdots x_n)$ is sufficient for θ
by Fisher/Neyman factorization