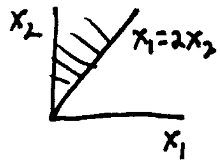
2.6#6
(15)



$$P(X_1<2X_2) = \int_0^\infty \int_0^{2x_2} e^{-x_1} e^{-x_2}\, dx_1\, dx_2$$
$$= \int_0^\infty (1-e^{-2x_2}) e^{-x_2}\, dx_2 = (-e^{-x_2} + \tfrac{1}{3} e^{-3x_2} \Big|_0^\infty = 2/3$$

$$\therefore\ P(X_1<X_2 \mid X_1<2X_2) = P(X_1<X_2\ \&\ X_1<2X_2)/P(X_1<2X_2)$$
$$= P(X_1<X_2)/P(X_1<2X_2) = (1/2)/(2/3) = 3/4$$

$$P(X_1<X_2<X_3<1) = \int_0^1 \int_0^{x_3} \int_0^{x_2} e^{-x_1} e^{-x_2} e^{-x_3}\, dx_3\, dx_2\, dx_1$$
$$= \int_0^1 \int_0^{x_3} e^{-x_3} e^{-x_2} (1-e^{-x_2})\, dx_2\, dx_1 = \int_0^1 e^{-x_3}(1-e^{-x_3} - \tfrac{1}{2} + \tfrac{1}{2} e^{-2x_3})\, dx_3$$

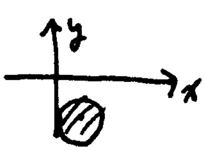
$$= -e^{-1}/2 + e^{-2}/2 - e^{-3}/6 + 1/6$$, and $P(X_3<1) = 1-e^{-1}$

so
$$P(X_1<X_2<X_3 \mid X_3<1) = P(X_1<X_2<X_3<1)/P(X_3<1) = (-3e^{-1}+3e^{-2}-e^{-3}+1)/6(1-e^{-1})$$

2.5#2
$$f_1(x_1) = \int_{x_1}^\infty 2e^{-x_1} e^{-x_2}\, dx_2 = 2e^{-x_1}(-e^{-x_2}\Big|_{x_1}^\infty = 2e^{-2x_1}$$
$$f_2(x_2) = \int_0^{x_2} 2e^{-x_1} e^{-x_2}\, dx_1 = 2e^{-x_2}(-e^{-x_1}\Big|_0^{x_2} = 2e^{-x_2}(1-e^{-x_2})$$

so $f_1(x_1) f_2(x_2) \neq f(x_1,x_2)$, so not indep.

2.5#7



$$(x-1)^2+(y+2)^2 \le 1 \Leftrightarrow x = 1 \pm \sqrt{1-(y+2)^2} \Leftrightarrow y = -2 \pm \sqrt{1-(x-1)^2}$$

$$f_2(y) = \int_{1-\sqrt{1-(y+2)^2}}^{1+\sqrt{1-(y+2)^2}} 1/\pi\, dx = \frac{2}{\pi}\sqrt{1-(y+2)^2}, \quad -3<y<-1$$

$$f_1(x) = \int_{-2-\sqrt{1-(x-1)^2}}^{-2+\sqrt{1-(x-1)^2}} (1/\pi)\, dy = \frac{2}{\pi}\sqrt{1-(x-1)^2}, \quad 0<x<2$$

$$f_1(x) f_2(y) = \frac{4}{\pi^2}\sqrt{(1-(y+2)^2)(1-(x-1)^2)} \neq \frac{1}{\pi}$$, dependent

3.1 #2 $M_X(t) = \left(\frac{2}{3} + \frac{1}{3}e^t\right)^9$ so $X \sim \mathrm{Bin}(9, \frac{1}{3})$ $\mu = 9\cdot\frac{1}{3} = 3$ $\sigma^2 = 9\cdot\frac{1}{3}\cdot\frac{2}{3} = 2$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = P(\underbrace{3-2\sqrt{2}}_{\sim .2} < X < \underbrace{3+2\sqrt{2}}_{\sim 5.8}) = P(1 \leq X \leq 5)$$

$$= \sum_{x=1}^{5} \binom{9}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{9-x}$$

3.2 #8 Note $X \sim \mathrm{Poiss}(\lambda)$ and $E(X) = \lambda$, so want λ least with

$P(X \geq 2) \geq .99 \Rightarrow P(X = 0 \text{ or } 1) \leq .01 \Rightarrow e^{-\lambda}(1+\lambda) \leq .01$

Can't give a closed-form sol'n to $1+\lambda = e^{\lambda}/100$, but $\lambda \approx 6.6$ works.

3.2 #12 $X \sim \mathrm{Poiss}(1)$, $E(X!) = \sum_{n=0}^{\infty} e^{-1}(1^n/n!)n! = e^{-1}\sum_{n=0}^{\infty} 1 = \infty$, doesn't exist