M631 Non-text problems Set 1

1. Call a subset $A$ of $\mathbb{N}$ cofinite provided $\mathbb{N} \setminus A$ is finite. Show that the set of all finite and cofinite subsets of $\mathbb{N}$ is an algebra, but is not a $\sigma$–algebra.

2. Call a subset $A$ of $\mathbb{R}$ co-countable provided $\mathbb{R} \setminus A$ is countable. Show that the set of all countable and co-countable subsets of $\mathbb{R}$ is a $\sigma$-algebra.

3. Describe the smallest algebra on $\mathbb{R}$ containing all intervals of the form $(a, b)$ where $a < b$.

4. Describe the smallest algebra on $\mathbb{R}$ containing all intervals of the form $[a, b]$ where $a < b$. 