Exercises 1

1. Suppose ${}^*\mathbb{R} \setminus \mathbb{R}$ is nonempty; prove that there are infinitesimals. Do not use saturation, you should be able to do this just using transfer of the properties of $\mathbb{R}$.

2. prove Corollary 3.1 from the lecture notes: If $x \approx y$ and $u \approx v$ then $x \pm u \approx y \pm v$. If x and u are finite then $xu \approx yv$

3. Prove the *underspill* property: If $A \subseteq^* \mathbb{R}$ is internal and contains arbitrarily small finite positive numbers, then it contains an infinitesimal.