Exercises 2

1. Prove the second part of the Prolongation result (p23 in the notes: Let $s_n \in {}^*\mathbb{R}$ for every $n \in \mathbb{N}$; then there is an internal sequence $\{t_n\}_{n\in {}^*\mathbb{N}}$ such that $s_n = t_n$ for every standard n.)

2. Suppose Ω is internal, and $f : \Omega \to NS({}^*\mathbb{R})$ is internal. Show that f is bounded by a standard integer.

3. Let $\{s_n\}_{n\in\mathbb{N}}$ be a sequence of real numbers. Prove that r is a limit point of $\{s_n\}_{n\in\mathbb{N}}$ if and only if there exists an infinite $k \in {}^*\mathbb{N}$ with $s_k \approx r$

4. Let $f : (a, b) \to \mathbb{R}, a < c < b$. Prove that the following is equivalent to the usual definition of "f has a local maximum at c on (a, b)":
$$(\forall x \approx c)^*f(x) \leq f(c)]$$

5. A family $\mathcal{F}$ of functions from an interval I to $\mathbb{R}$ is (uniformly) equicontinuous provided:
$$\forall \epsilon > 0 \exists \delta > 0 \forall f \in \mathcal{F}\ \forall x, y \in I, |x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon.$$
Find a simple nonstandard equivalent, and prove it is equivalent.