Exercises 2

1. Prove Theorem 5.1(4) from the notes: $\{s_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence if and only if for every infinite $m, n \in^* \mathbb{N}, s_m \approx s_n$

2. Prove Theorem 5.1(4) from the notes: r is a limit point of $\{s_n\}_{n\in\mathbb{N}}$ if and only if there exists an infinite $k \in^* \mathbb{N}$ with $s_k \approx r$

3. Let $f, \phi : I \to \mathbb{R}$, where I is an interval; prove that TFAE:

   (a) $(\forall\epsilon > 0)(\exists\delta > 0)(\forall x, y \in I)[(0 \neq |y - x| < \delta) \Rightarrow (|\frac{f(y)-f(x)}{y-x} - \phi(x)| < \epsilon)]$

   (b) $(\forall x \neq y \in^* I)[(y \approx x) \Rightarrow (\frac{^*f(y)-^*f(x)}{y-x} \approx^* \phi(x))]$

   (cf the definition of *uniformly diffentiable* in the notes)

4. Let $f : (a, b) \to \mathbb{R}, a < c < b$. Prove that the following is equivalent to the usual definition of "f has a local maximum at c on (a, b)":

$$(\forall x \approx c)^*f(x) \leq f(c)]$$