

Exercises 3

The first 3 exercises refer to the posted notes on topological spaces. They should be easy; if not, you're doing them wrong!

1. Notes, Exercise 1.1 Part 4
2. Notes, Exercise 1.4
3. Notes, Exercise 2.2

The next 2 are more ambitious. Try to do at least one. The second one generalizes a result in the notes.

4. A *topological group* is a group (G, \cdot, e) which is also a topological space, and such that multiplication $(x, y \mapsto x \cdot y)$ and inversion $(x \mapsto x^{-1})$ are continuous functions. For sets $A, B \subseteq G$ write

$$A \cdot B = \{a \cdot b : a \in A, b \in B\}.$$

Prove that for any $x \in G$, $x \cdot \text{monad}(e) = \text{monad}(e) \cdot x = \text{monad}(x)$.

5. Let G be as in the last exercise. Show that if K is a compact subset of G and C is a closed subset of G then $K \cdot C$ is closed.