Exercises 4

These exercises refer to the posted notes on measure spaces. The first two should be easy; the last one is slightly tricky to get all the details right.

1. (see page 5 of the notes)Prove:
   - If X is σ-compact, $X = \bigcup_n X_n$ with X_n compact, then $NS({}^*X) = \bigcup_n^* X_n$.
   - If X is a complete metric space then $NS({}^*X) = \bigcap_n \bigcup_{x \in X}^* B_{1/n}(x)$
2. Verify Example 1.1 on page 5 of the notes
3. Prove that if $(X, \mathcal{A}_L\ , \mu_L)$ is a Loeb measure space then either it is finite (ie, $\mu_L(X) < \infty$) or it is not σ-finite. (see pg. 3 of the measure theory notes.)
4. We know from standard measure theory that any infinite sigma-algebra is uncountable. (If you're planning to take the analysis qual, you should convince yourself you know how to prove this!) Prove this related result: if $\mathcal{A}$ is an externally infinite internal algebra on a set X, then it isn't (externally) a $\sigma-$algebra. What can you say if it is externally *finite*?