1. a. Find the transition matrix $P_{T \leftarrow S}$ from $S = \{[1 \ 1 \ 1], [1 \ 2 \ 3], [1 \ 0 \ 1] \}$ to $T = \{[0 \ 1 \ 1], [1 \ 0 \ 0], [1 \ 0 \ 1] \}$, where these are two bases of $\mathbb{R}^3$.

b. Use the result of part (a) to find the coordinates of $[-1 \ 4 \ 5]$ with respect to the second basis.


3. Define $L : \mathbb{R}^3 \to \mathbb{R}^3$ by $L([a, b, c]) = [a + b + c, a - c, 0]$.
(a) What is $M(L)$ with respect to the standard basis?
(b) Let $S = \{[2, 1, 0], [1, 1, 0], [0, 1, 2] \}$ be another basis for $\mathbb{R}^3$. What is $M(L)$ with respect to $S$?
(c) If $[v]_S = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, what is $[L(v)]_S$? What is $L(v)$?
(d) Compute $\ker L$.

4. Let $L : V \to V$ be a linear transformation from a vector space $V$ to itself. Prove that $W = \{ v \in V \mid L(v) = 7v \}$ is a subspace of $V$.


6. Let $P$ be an invertible $n \times n$ matrix. Show that $L(A) = P^{-1}AP$ defines a linear transformation from $M_{nn}$ to $M_{nn}$.

7. Compute the determinant of $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 0 & 2 & -1 \\ -1 & 2 & 0 & 7 \\ -1 & 3 & 2 & 100 \end{bmatrix}$.

Answer to 3(b): $\begin{bmatrix} 1 & 1 & 5 \\ 1 & 0 & -7 \\ 0 & 0 & 0 \end{bmatrix}$

Answer to 3(c): $L(v) = [2 \ 5 \ 0]$