Math 411 **HOMEWORK #10**

Due Wednesday, April 9, 2014

1. Let A be a 2×2 matrix over a field F and suppose that $A^2 = 0$. Show that $\det(cI - A) = c^2$. [Do not use advanced theorems about eigenvalues and similar matrices.]

2. An n×n matrix over a field F is *skew-symmetric* if $A^t = -A$. If A is skew-symmetric with complex entries and n is odd, prove that $\det A = 0$

3. An n×n matrix over a field F is *orthogonal* if $A^t A = I$. If A is orthogonal, show that $\det A = \pm 1$. Give an example of an orthogonal matrix with determinant -1.

4. Let A be an n×n matrix with block form

$$\begin{pmatrix} A_1 & 0 & \dots & 0 \\ 0 & A_2 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & A_k \end{pmatrix}$$

where A_i is an $r_i \times r_i$ matrix. Show that $\det A = (\det A_1) \cdots (\det A_k)$.

5. Let A be an n×n matrix over a field F. Prove that there are at most n distinct scalars $c \in F$ such that $\det(cI - A) = 0$.