Math 411 **HOMEWORK #9**

Due Wednesday, April 2, 2014

1. Let $V = \mathbb{C}_2$ with the standard inner product. Let T be the linear transformation defined by $T\langle 1, 0\rangle = \langle 1, -2\rangle$, $T\langle 0, 1\rangle = \langle i, -1\rangle$. Find $T^*\langle x_1, x_2\rangle$.

2. Let V be a finite dimensional inner product space and $T \in L(V, V)$. Show that the range of T^* is the orthogonal complement of the kernel of T.

3. Let V be a finite dimensional inner product space and $T \in L(V, V)$. If T is invertible, show that T^* is invertible and $(T^*)^{-1} = (T^{-1})^*$.

4. Let V be the vector space of all real-valued differentiable functions f on the interval $[0, 1]$ such that $f(0) = f(1) = 0$. Define an inner product on V by

$$(f, g) = \int_0^1 f(t)g(t)\,dt.$$

Let D be the differentiation operator and compute its adjoint D^*.