1. a. Let $G = GL_3(\mathbb{F}_p)$ denote the multiplicative group of all $3 \times 3$ nonsingular matrices with entries in the field of $p$ elements $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$. Show that the set of all matrices of the form
\[
\begin{pmatrix}
1 & a & b \\
0 & 1 & c \\
0 & 0 & 1
\end{pmatrix}
\]
where $a, b, c \in \mathbb{F}_p$, is a Sylow $p$-subgroup of $G$. [Hint: count.]

b. Is it the only Sylow $p$-subgroup of $G$?

2. Let $G$ be a group and let
\[
G = G_0 \supseteq G_1 \supseteq G_2 \supseteq \ldots
\]
and
\[
\{e\} = Z_0 \subseteq Z_1 \subseteq Z_2 \subseteq \ldots
\]
be its descending and ascending central series. Prove:

a. If $Z_m = G$, then for all $i \leq m$, $G_i \subseteq Z_{m-i}$.
b. If $G_m = \{e\}$, then for all $j \leq m$, $G_{m-j} \subseteq Z_j$.

In particular, $G_m = \{e\}$ if and only if $Z_m = G$.


4. Let $G = \mathbb{Z}_{12}$. Find a composition series for $G$ and compute the Jordan-Hölder components.