


4. Let $C$ be a category and $\{A_i\}_{i \in I}$ a set of objects in $C$. Show that if the product exists, it is unique (up to isomorphism).

5. **Induced Homomorphism Theorem.**

\[
\begin{align*}
X & \quad \xrightarrow{\phi} \quad K \quad \xrightarrow{\alpha} \quad M \quad \xrightarrow{\zeta} \quad N \quad \xrightarrow{\beta} \quad C \quad \xrightarrow{\psi} \quad Y \\
\downarrow & \quad & \downarrow \psi & \quad \downarrow & \quad & \downarrow & \\
0 & \quad \xrightarrow{} \quad K & \quad \xrightarrow{} \quad M & \quad \xrightarrow{} \quad N & \quad \xrightarrow{} \quad C & \quad \xrightarrow{} \quad 0
\end{align*}
\]

Assume that the row above is exact. [You may assume you are working in $Ab$ but the theorem is true in $Gp$ and in any abelian category.]

a. Prove that $\phi$ “factors through $K$” (i.e. there exists $\phi' : X \to K$ such that $\phi = \alpha \phi'$) if and only if $\zeta \phi = 0$.

b. Prove that $\psi$ “factors through $C$” (i.e. there exists $\psi' : C \to Y$ such that $\psi = \psi' \beta$) if and only if $\psi \zeta = 0$. 