QUIZ 1

(5) 1. One of the diameters of a sphere has endpoints $(-1,2,0)$ and $(3,0,4)$. What is the equation of the sphere?

Center = midpoint of diam = \(\left(-\frac{1+3}{2}, \frac{2+0}{2}, \frac{0+4}{2}\right) = (1,1,2)\)

Radius = \(\frac{1}{2}\) diam = \(\frac{1}{2}\sqrt{(3-(-1))^2 + (0-2)^2 + (4-0)^2} = \frac{1}{2}\sqrt{16 + 4 + 16} = \frac{6}{2} = 3\)

Equation: \((x-1)^2 + (y-1)^2 + (z-2)^2 = 3^2\)

(5) 2. Find the value(s) of \(a\) such that the vector \(<a,-1,1>\) is perpendicular to the vector \(<1,2,4>\), or show that no such \(a\) exist.

\(<a,-1,1> \perp <1,2,4> \text{ when} \)

\(0 = <a,-1,1> \cdot <1,2,4> = a - 2 + 4 = a + 2\)

so \(\boxed{a = -2}\) is the only solution.

(5) 3. True-False. Just write T or F; no justification is required.

a. For any vectors \(\vec{u}\) and \(\vec{v}\), \(\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}\).

\(\boxed{T}\)

b. For any vectors \(\vec{u}\) and \(\vec{v}\), \(\vec{u} \times \vec{v} = \vec{v} \times \vec{u}\).

\(\boxed{F}\) since \(\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}\), not \(\vec{u} \times \vec{v} = \vec{v} \times \vec{u}\)

So, \(\vec{u} \times \vec{v} \neq -\vec{v} \times \vec{u}\) is a counterexample.

c. For any vectors \(\vec{u}, \vec{v}\) and \(\vec{w}\), \((\vec{u} + \vec{v}) \times \vec{w} = \vec{u} \times \vec{w} + \vec{v} \times \vec{w}\).

\(\boxed{T}\)

d. For any vectors \(\vec{u}, \vec{v}\) and \(\vec{w}\), \((\vec{u} \times \vec{v}) \times \vec{w} = \vec{u} \times (\vec{v} \times \vec{w})\).

Counterexample from class: \((\vec{e}_1 \times \vec{e}_2) \times \vec{e}_1 = \vec{e}_2 \times \vec{e}_1 = \vec{e}_3\) but \(\vec{e}_1 \times (\vec{e}_2 \times \vec{e}_1) = \vec{e}_1 \times \vec{e}_3 = \vec{e}_2\)

e. For any vectors \(\vec{u}\) and \(\vec{v}\) and any scalar \(k\), \(\vec{u} \times (k\vec{v}) = k(\vec{u} \times \vec{v})\).

\(\boxed{T}\)

The "True" statements were all discussed in class.

See your notes.
4. Find parametric equations (vector or scalar - your choice) for the line that is perpendicular to the plane $2x - 5z = 10$ and contains the point $(4, -3, 7)$.

The plane has normal $\vec{N} = \langle 2, 0, -5 \rangle$ from the coefficients of the equation. So use $\langle 2, 0, -5 \rangle$ as the direction vector. Position vector $\vec{a} = \langle 4, -3, 7 \rangle$ since $(4, -3, 7)$ is on the line.

So vector parametric: $\vec{r}(t) = \langle 4, -3, 7 \rangle + t \langle 2, 0, -5 \rangle$

or scalar parametric:

$x(t) = 4 + 2t$
$y(t) = -3$
$z(t) = 7 - 5t$

5. Find an equation for the plane that contains the x-axis and the point $(1, 2, 3)$.

$\vec{e}$ is on the plane, since $\vec{e}$ is along the x-axis.

Also $\langle 1, 0, 3 \rangle$ is on the plane, since $(0, 0, 0)$ and $(1, 2, 3)$ are both on the plane.

So we can use $\vec{e} \times \langle 1, 2, 3 \rangle$ as a normal vector for the plane.

$N = \langle 1, 0, 3 \rangle \times \langle 1, 2, 3 \rangle = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
1 & 0 & 3 \\
1 & 2 & 3
\end{vmatrix} = \langle -6, -3, 2 \rangle$

$(0, 0, 0)$ is on the plane, so the equation is

$0(x - 0) - 3(y - 0) + 2(z - 0) = 0$

or $-3y + 2z = 0$