EXAM RULES:

- No books, notes, or other study aids except for a calculator provided by the math department. You may not use your own calculator.
- Phones must be turned off. If your phone rings during the test, you will automatically lose 5 points.
- Sending or receiving text messages during the exam constitutes cheating and will be treated as such.
- No restroom breaks until you have turned in your exam.
- On sections where you have a choice, clearly mark the problem you want graded, otherwise, this choice will be made for you.
1. (6 pts) Give formal definitions of the following. (For (a) “formal” means involving limits.)

(a) A continuous function.

(b) A rational function.

2. (3 pts) Express the following in terms of the functions $\sin \theta$, $\cos \theta$, $\sin^2 \theta$, $\cos^2 \theta$.

(a) $\tan \theta =$

(b) $\sec \theta =$

(c) $1 =$

3. (2 pts) Prove the following:

Fact: $\sec^2 \theta = 1 + \tan^2 \theta$.

Proof:

\footnote{Note: the proof is only worth two points, so don’t spend too long on it.}
4. (10 points) **Limits.** Answer question (a) and then answer either (b) or (c).

(a) (6 pts) If \( f(t) = \frac{t^3 + 3t^2 - 2}{(-5t^3 + 100)} \), find the following limits:

(i) \( \lim_{t \to 0} f(t) \)

(ii) \( \lim_{t \to -\infty} f(t) \)

(b) (4 pts) Let \( h(x) = \frac{p(x)}{q(x)} \) where \( p(x) = \sqrt{x} - 8 \) and \( q(x) = x - 64 \). Find the limit of \( h(x) \) as \( x \) approaches a point of discontinuity.

(c) (4 pts) Compute the limit \( \lim_{t \to \infty} \frac{\sin t}{t} \). *(Hint: Recall that the value of \( \sin t \) is always between -1 and 1.)*
5. (12 pts) **Differentiation.** Find the derivative of each function. For the last one, also find the *second* derivative.

(a) \( g(t) = (2e)^{t^{0.3}} \)

(b) \( h(\theta) = \sqrt{\cot(x)} + 7x^3 \)

(c) \( u(x) = (3x^3 \ln\left(\frac{x^2}{\pi}\right) \)

(d) Let \( f(x) = e^{-x^2} \). Compute the *second* derivative, \( f''(x) \).
6. (7 pts) **Implicit Differentiation.** Solve either (a) or (b).

(a) Find the equation of the line tangent to the unit circle \(x^2 + y^2 = 1\) at the point where the angle is \(7\pi/4\).

(b) Find the equation of the line tangent to the curve \(y^3 + 2x^2y - 8y = x^3 + 19\) at the point \((x, y)\) where \(x = 2\).
7. (12 pts) **Applications of Extrema.** Solve either (a) or (b).

(a) To pass a lifeguard test at Waikiki beach, a lifeguard must get from point $A$ at the water’s edge to a buoy in the water at point $B$ in the shortest possible time. Suppose the buoy is exactly 30 m off-shore. Let $C$ be the point on the beach closest to the buoy and suppose $C$ is exactly 40 m from where the lifeguard stands at point $A$. The lifeguard wants to know the best point $P$ on the beach at which to jump in and start swimming. Suppose her running speed is 3 meters per second and her swimming speed is 1 meter per second.

(i) (3 pts) Draw a picture of the scene and label it.

(ii) (4 pts) Find a function $T(x)$ that gives the time it takes to get from point $A$ to point $B$, given an input value $x$ (for example, you might decide that $x$ should represent the distance from $C$ to $P$).

(iii) (5 pts) Find the value of $x$, and hence the point $P$, that will minimize the time it takes to reach the buoy.
(b) A farmer has 2,400 m of fencing. He wants to enclose a rectangular field bordering a river, with no fencing needed along the river.

(i) (3 pts) Draw a picture of the scene and label it.

(ii) (4 pts) Find a function $A(x)$ that gives the area of the field for any given input value $x$ (for example, you might decide that $x$ should represent the width of the field).

(iii) (5 pts) Find the maximum area.
(c) (8 pts) **Related Rates.** Solve either (a) or (b).

(a) A sand storage tank used by the highway department is leaking. As the sand leaks out, it forms a conical pile. The radius of the base of the pile increases at a rate of 2 cm per min. The height of the pile is always twice the radius of the base. Find the rate at which the volume of the pile is increasing the instant the radius of the base is 8 cm.

(b) At noon, a Somali pirate ship $S$ is 40 nautical miles due west of an oil tanker $T$. Ship $S$ is sailing west at 19 knots and the tanker $T$ is sailing north at 22 knots. How fast (in knots) is the distance between the ships changing at 6pm? (Note: 1 knot is a speed of 1 nautical mile per hour).