1. (2 pts) Give formal definitions of the following. (For the first two “formal” means involving limits.)
   
   (a) A continuous function.
   
   (b) The derivative of a function.
   
   (c) A polynomial of degree \( n \).
   
   (d) For \( a > 0, a \neq 1, \) and \( x > 0 \), define \( y = \log_a(x) \) (in terms of exponents).

2. (2 pts) Express the following in terms of sines and cosines:
   
   (a) \( \tan(\theta) = \)
   
   (b) \( \cot(\theta) = \)
   
   (c) \( \sec(\theta) = \)
   
   (d) \( \csc(\theta) = \)

3. (2 pts) Prove the following

   **Fact:** \( \frac{d}{dt} \ln \left| \sec(t) + \tan(t) \right| = \sec(t) \)

   **Proof:**
4. (2 pts) **Limits.**

(a) If

\[ f(x) = \frac{2x^4 + x^3 - 3x + 1}{-7x^4 - 4x^5 + 2}, \]

find the limit of \( f(x) \) as \( x \) approaches \( \infty \).

(b) Let \( f(x) = p(x)/q(x) \) where \( p(x) = x^2 - x - 6 \) and \( q(x) = x - 3 \). Find the limit of \( f(x) \) as \( x \) approaches a point of discontinuity.

5. (2 pts) Find the equation of the line tangent to the unit circle \( x^2 + y^2 = 1 \) at the point where the angle is \(-5\pi/6\).
6. (5 pts) Find the first derivative of each of the following functions. For the last one, also find the second derivative.

(a) \( f(x) = 2\sqrt{x} \)

(b) \( g(t) = t^{0.3} + \cos(\pi/4) \)

(c) \( h(\theta) = \sqrt[3]{\tan(\theta)} + \frac{\sin(\theta)}{\theta} \)

(d) \( u(x) = (2x)^3 \left[ x^4 - 5 \ln\left(\frac{x^2}{\pi}\right) \right] \)

(e) Let \( f(x) = e^x \sin(2x) + \sqrt{1 + x^2} + e^x \). Compute the second derivative, \( f''(x) \).
7. (5 pts) **Applications.** Solve any *one* of the following three problems, (a), (b), or (c).

(a) **Pigeon Flight.** Homing pigeons avoid flying over large bodies of water, preferring to fly around them instead. Assume that a pigeon released from a boat 1 mi from the shore of a lake flies first to point $P$ on the shore and then along the straight edge of the lake to reach its home at $L$. If $L$ is 2 mi from point $A$, the point on the shore closest to the boat, and if a pigeon needs $10/9$ as much energy per mile to fly over water as over land, find the location of point $P$, which minimizes energy used.

(b) **Water Level.** A trough has a triangular cross section. The trough is 6 feet across the top, 6 feet deep, and 16 feet long. Water is being pumped into the trough at the rate of 4 cubic feet per minute. Find the rate at which the height of water is increasing at the instant that the height is 4 feet.
(c) **Minimizing Area.** You have just been hired as a consultant by The Coca-Cola Company. Your first assignment is to decide whether the current can design is the best, most efficient one possible. Of course, they will continue to produce cans of Coke containing 12 fluid ounces, which is equivalent to 355 cm$^3$. That is, the volume of the can must be 355 cubic centimeters.

(i) (3 pts) Find the dimensions (radius of base and height) of the can which uses the least amount of aluminum. *(Hint: volume of a cylinder is equal to the area of the base times the height.)*

(ii) (1 pt) Of course, soft drink manufacturers account for other things besides the cost of aluminum, and current can designs typically have a base radius of approximately 3.25 cm and a height of about 12 cm. Given these dimensions, how much aluminum do cans currently use? (You can ignore the fact that the dimensions of such cans don’t appear to hold 12 ounces.)

(iii) (1 pt) If Coca-Cola decides to adopt your new can design, how much aluminum will they save per can?