6. (5 pts) Find the first derivative of each of the following functions. For the last one, also find the second derivative.

(a) \( f(x) = 2\sqrt{x} \)

**Solution:** Recall, if \( f(x) = a^{g(x)} \) for some constant \( a > 0 \) and function \( g(x) \), then \( f'(x) = \ln(a)g'(x)a^{g(x)} \). So, if \( f(x) = 2\sqrt{x} \), then (with \( a = 2 \) and \( g(x) = \sqrt{x} \))

\[
f'(x) = \ln(2)g'(x)a^{g(x)} = \ln(2) \left( \frac{1}{2}x^{-1/2} \right) 2\sqrt{x}.
\]

\( \square \)

(b) \( g(t) = t^{0.3} + \cos(\pi/4) \)

**Solution:** Note that \( \cos(\pi/4) \) is just a constant \( (\sqrt{2}/2) \) so its derivative is 0. By the power rule, then,

\( g'(t) = 0.3t^{-0.7}. \)

\( \square \)

(c) \( h(\theta) = \sqrt{\tan(\theta)} + \frac{\sin(\theta)}{\theta} \)

**Solution:** We have \( h(\theta) = [\tan(\theta)]^{1/3} + \theta^{-1} \sin(\theta) \). Therefore, by the chain rule and product rule,

\[
h'(\theta) = (1/3)[\tan(\theta)]^{-2/3}\left[ \frac{d}{d\theta} \tan(\theta) \right] + (-\theta^{-2}) \sin(\theta) + \theta^{-1} \cos(\theta).
\]

Now recall that \( \frac{d}{d\theta} \tan(\theta) = \sec^2(\theta) \), so the answer is

\[
h'(\theta) = (1/3)[\tan(\theta)]^{-2/3} \sec^2(\theta) - \frac{\sin(\theta)}{\theta^2} + \frac{\cos(\theta)}{\theta}.
\]

\( \square \)

(d) \( u(x) = (2x)^3 \left[ x^4 - 5 \ln\left( \frac{x^2}{\pi} \right) \right] \)

**Solution:** Recall, if \( f(x) = \ln[g(x)] \) then

\[ f'(x) = \frac{g'(x)}{g(x)}. \]

If \( f(x) = \ln\left( \frac{x^2}{\pi} \right) \), then (with \( g(x) = \frac{x^2}{\pi} \)),

\[
f'(x) = \frac{g'(x)}{g(x)} = \frac{2x/\pi}{x^2/\pi} = \frac{2}{x}.
\]

Therefore, by the product rule and chain rule,

\[
u'(x) = 3(2x)^2(2) \left[ x^4 - 5 \ln\left( \frac{x^2}{\pi} \right) \right] + (2x)^3 \left[ 4x^3 - 5 \left( \frac{2}{x} \right) \right].
\]

\( \square \)

(Please note: many of you wrote the first factor \( 3(2x)^2(2) \) as \( 6x^2(2) \) or \( 12x^2 \), which is incorrect. If you want to be especially careful not to make such a mistake again, consider re-writing the first factor of the original function as \( (2x)^3 = 2^3x^3 \), and then \( 2^3 = 8 \) is just a constant out in front of the function. (As a nice consequence, there is one less application of the chain rule when you solve it this way.)
(e) Let \( f(x) = e^x \sin(2x) + \sqrt{1 + x^2} + e^\pi \). Compute the second derivative, \( f''(x) \).

**Solution:** First note that \( e^\pi \) is a constant, so \((e^\pi)' = 0\). Compute the first derivative using the product rule and the chain rule as follows:

\[
f'(x) = e^x \sin(2x) + 2e^x \cos(2x) + \frac{1}{2}(1 + x^2)^{-1/2}(2x).
\]

Compute the second derivative by taking the derivative of (1) (the extra brackets in my answer will help you decipher which of the three terms in the sum above correspond to which part of my answer)

\[
f''(x) = [e^x \sin(2x)]' + [2e^x \cos(2x)]' + \left[ \frac{1}{2}(1 + x^2)^{-1/2}(2x) \right]' \\
= [e^x \sin(2x) + 2e^x \cos(2x)] + [2e^x \cos(2x) + 2e^x (- \sin(2x))](2x) + \left[ \frac{-1}{4}(1 + x^2)^{-3/2}(2x)(2x) + \frac{1}{2}(1 + x^2)^{-1/2}(2) \right].
\]

You don’t have to simplify, but if you do, the answer will look something like this:

\[
f''(x) = e^x[4 \cos(2x) - 3 \sin(2x)] - x^2(1 + x^2)^{-3/2} + (1 + x^2)^{-1/2}.
\]

or, simplifying even more, like this:

\[
f''(x) = e^x[4 \cos(2x) - 3 \sin(2x)] + \frac{1}{(1 + x^2)^{3/2}}.
\]

\[\square\]